WRITTEN SUBTRACTION ALGORITHMS

Time for a change?

Ian Thompson

Mental calculation is concerned with flexibility and the use of methods which are meaningful to the person doing the calculating. This suggests that we need to rethink the written algorithms that we teach – particularly those for subtraction. This article considers three written subtraction algorithms that are different from the methods that are usually taught in this country – decomposition and equal additions – and which are more closely related to the mental strategies that children in this country actually use for this operation.

INTRODUCTION

In an earlier article in this journal (Thompson, 1995) I discussed the feasibility of narrowing the gap between children's idiosyncratic mental calculation algorithms and the traditional, standard written algorithms. I looked at a range of written procedures for addition produced by children who had not been taught the standard methods, and concluded by recommending the following 'user-friendly' addition algorithm:

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  256
 +178
  300
  120
   14
  434
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Since this earlier article was published, substantial developments have taken place in the area of mental arithmetic. International comparisons appear to suggest that many of the countries that perform better than us on formal paper and pencil tests in number have a more structured approach to the teaching of mental calculation than we have in this country. Assessment of 'mental arithmetic' has become an intrinsic part of National Curriculum Tests, and the Qualifications and Curriculum Authority is currently producing guidance for teachers on the teaching of mental strategies for calculation.

In addition to this, four recent innovations in primary mathematics education – the National Numeracy Project, the Improving Primary Mathematics initiative (Barking and Dagenham), the Hamilton Maths Project, and the Gatsby-funded Mathematics Enhancement Programme (Primary) – have all placed great emphasis on the importance of developing mental calculation strategies in young children. For example, the National Numeracy Project has made mental calculation one of the linchpins of its Framework Document. It has developed a detailed structure for the teaching of mental calculation, and has recommended delaying the teaching of vertical pencil and paper algorithms for the basic operations until Key Stage 2.

All of these factors point to the substantial importance that is being accorded to this key aspect of number work. Having already raised some questions about the appropriateness of the standard algorithm for addition, I would like, in this article, to initiate a debate on possible alternative algorithms for subtraction which are more related to children's mental calculation strategies that those currently taught.

THE NEED FOR CHANGE

To a large extent the hallmark of mental calculation is *flexibility*. Few adults use the same strategy each time they perform a calculation in their head. The methods they do use often depend on the specific numbers involved and on their confidence in their own number fact knowledge. The development of this ability to select an appropriate mental algorithm must be an important aim in the teaching of numeracy. A number curriculum for the next millennium should incorporate a structured approach to the teaching of mental calculation. Children need to develop a range of skills which they can combine to form a suitable strategy for dealing with any arithmetic calculation.

The teaching of this number curriculum, with its focus on mental calculation, would need to be based on research into the methods that children have been shown to use for carrying out calculations in their head. Vertical written methods for the basic operations would not be taught in Key Stage 1 – a key recommendation of the National Numeracy Project. Another important aim of this curriculum would be the development of children's proficiency in mental calculation strategies to such an extent that they would be able to add or subtract any pair of two-digit numbers in their heads with confidence and accuracy, resorting only occasionally to pencil and paper support. In order to cope with the subtraction of three-digit numbers, they would need to have access to a written algorithm that *supported* their mental methods. But do such algorithms exist?

Partitioning

Researchers in Holland (Beishuizen, 1993), South Africa (Murray and Olivier, 1989), Britain (Thompson, 1997) and Australia (Cooper et al, 1996) have all suggested that the most common mental strategies for two-digit subtraction involve the partitioning of numbers into multiples of ten and single-digit numbers – a procedure similar to that adopted in the addition example illustrated above. So, 56 – 32 would either be calculated as '50 – 30 is 20 … 6 – 2 is 4 … so the answer is 24', or as '56 – 30 is 26 … 26 – 2 is 24'. In the first example both numbers are partitioned ('double partitioning'), whereas in the second example it is only the number to be subtracted that is affected ('single partitioning').

A common strategy for the mental subtraction of a one-digit number from a two-digit number involves 'bridging to ten', and is illustrated by Mark in his calculation of 24 – 7: '17 … When I've got 24 and I take away 4 it makes 20 … and I know that 3 and 17 makes 20 … so I have 3 left to make 17'. Mark has been influenced by the units digit of the 24 in his decision to subtract four first rather than try to subtract seven in one fell swoop. Doing this leaves him with a further three to take away, but as it is from a nice round multiple of ten he can use his knowledge of 'complements within 20'.

Developing, and then extending, the separate skills involved in Mark's strategy would enable children to cope with more complex subtractions, such as 54 – 37, in their heads. These skills would include the ability to:

- partition any two-digit number (37 is 30 and 7);
- partition any single-digit number in a variety of ways (7 can be split into a 4 and a 3);
- subtract a single-digit number from any multiple of ten (50 – 3 is 47);
- subtract a two-digit multiple of ten from another (70 – 30 is 40);
- subtract a three-digit multiple of a hundred from another (700 – 300 is 400).

Double partitioning

By utilising these specific skills children should be able to execute the following subtraction algorithm with understanding. Although horizontal algorithms could be illustrated (and taught), I have chosen to show vertical methods as they then resemble the layout used for the decomposition or equal additions methods currently taught in schools. In order to enable readers to interpret the written method, a two-digit subtraction is shown first, and a 'running commentary' is included to illustrate the thinking involved:

65	
–27	'60 take 20 is 40 …
40	… 5 take 7 … If I take 5 away that leaves me with 2 more to take away …
– 2	… Write down the 2 that I still have to take away.'
38	

With three-digit numbers it is not much more difficult:

754	
– 286	'700 take 200 is 500 …
500	50 take 80! … If I take away 50 of the 80.
–30	… then I've still got 30 more to take away.
470	4 take 6!… If I take away 4 of the 6…
– 2	… then I still have 2 more to take away.
468	… so the answer is 468.'

Single partitioning

In this method, only the number to be subtracted (the 'subtrahend') is partitioned:

65	
–27	
45	'Take 20 from the 65 to get 45 …
– 5	… I've still got 7 to take away, so take 5 first …
40	… that gives a nice round number
– 2	… Now take the remaining 2 …
38	… that leaves me with 38.'

754	
– 286	'754 take 200 is 554 …
554	… 80 to take, so I'll take 50 first
–50	
504	then I'll take the 30
–30	
474	just 6 to take now, so if I take 4
– 4	
470	that takes me to a multiple of ten
– 2	and taking the remaining 2
468	gives 486.'

The *single partition* procedure is the preferred method taught to children in the Netherlands. It is known as the N10 strategy ('one *number* has the *tens* part of the other subtracted first'). These methods may well appear complicated to adults weaned on decomposition or equal additions, but these adults are not likely to possess the range of mental strategies that children brought up on the National Numeracy Strategy will have developed.

In the Netherlands much work is done on introducing the children to the *empty number line* (see Bramald, R in press), as it stimulates the use of personal mental strategies, offers a natural and transparent representational model for a variety of problem types, and acts as a support for students' thinking, helping them keep track of the intermediate stages of their working.

Complementary addition

Real-life situations that can be solved by subtraction generally fall into one of two types: those which involve the removal of some objects from a collection, or those which involve some form of comparison between collections. When young children start school they have usually had substantial experience of the 'take-away' aspect of subtraction, and it was for this reason that some primary schemes in the 1970s began work on subtraction with the 'difference' aspect in order to compensate for this. The partitioning algorithms described above relate quite closely to 'take-away' situations. However, there is an alternative algorithm which embodies the 'difference' aspect of subtraction, and makes use of many of the skills which children following a structured approach to mental calculation should possess.

This procedure has been known and used informally for many years, and was once called 'shopkeeper arithmetic' in the days when shop assistants would count out the change from a purchase one coin at a time. It is currently known in this country as 'complementary addition', and it lends itself to being formalised for three- or four-digit written calculations while at the same time allowing a substantial amount of flexibility on the part of the user. The procedure utilises the following mental calculation skills:

- finding the complement in ten of a single-digit number (4 is the complement of 6);
- working out the complement in 100 of any multiple of ten (30 is the complement of 70);
- calculating what you need to add on to a multiple of 100 to get any higher multiple (from 200 to 700 is 500), or,
- (at a more sophisticated level) calculating what you add on to a multiple of 100 to get any three-digit number (from 300 to 576 is 276).

Developing these skills will help children to use the algorithm with understanding.

In the following three-digit examples the numbers immediately to the right of the algorithm constitute the running total, whereas the answer is the sum of the 'jumps':

754		
– 376		
4	380	'376 to 380 is 4 …
20	400	… 380 to 400 is 20 .
300	700	… 400 to 700 is 300
54	754	… 700 to 754 is 54.'
378		

As children gain more experience of the method they might wish to shorten it to:

754		
– 376		
4	380	376 to 380 is 4 …
20	400	.. 380 to 400 is 20 .
354	754	.. 400 to 754 is 354.
378		

An even more concise form of recording might read:

754		
– 376		
24	400	376 to 400 is 24 …
354	700	.. 400 to 754 is 354.
378		

It is not essential that the running totals which signal the successive stages of the calculation – 380, 400, 700, 754 – are written down. They are included to help the individual keep track. An alternative approach is to verbalise the subtotals while writing down (horizontally or vertically) the numbers that have been added on at each stage. This algorithm is apparently taught formally in some European countries but not, to my knowledge, in Britain. It is a holistic method which treats the numbers as whole entities rather than as the sum of separate hundreds, tens and ones. It also demands, and builds upon, a good standard of mental calculation facility.

CONCLUSION

One might argue that children should be encouraged to develop their own idiosyncratic written methods or that a greater emphasis on mental calculation in our number curriculum should render written algorithms redundant. To a certain extent I agree with these statements, but, given the obsession with standard algorithms in this country, I see the suggestions in this article as an interim measure; as a first attempt to wean teachers and text-book writers off written algorithms that are not at all related to mental methods. The algorithms described in this article have the added advantage of modelling the two main categories of subtraction problem that young children are likely to meet in their everyday life and in school; those involving comparison and those involving taking away. Consequently, by helping children develop the written procedures described in this article, teachers will be equipping their pupils with methods appropriate for either situation. Of course, it is not necessary for *all* children to learn all three procedures, particularly as they demand that the user has a good understanding of the underlying mathematics. What *is* important, however, is that all pupils develop a written procedure for three-digit subtraction that makes sense to them, can be understood by others, and that provides the best match with their own idiosyncratic mental calculation strategies.

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