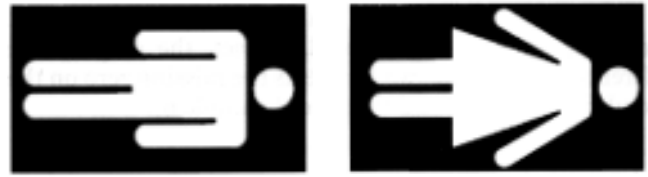


# United we stand; divided we fall



by Ian Thompson



I understand division; or at least I thought I did. I mean, after all, I've taught it to primary children, secondary students, PGCE trainees, parents, teachers and National Numeracy Strategy consultants over the last few decades without any apparent problems. It was only when I decided to think about doing some research in this area that I realized that I understood much less than I had originally thought. The following paragraphs recount some of the difficulties, contradictions and confusions that I encountered during and after my preliminary reading.

## Short or Long?

Everyone knows about 'long division': but what is it? (Before reading on, think about how you would differentiate between long and short division). There seem to be at least two different interpretations of what 'long' means in this context. The NNS *Framework for Teaching Mathematics* (1999) interprets short division in terms of those situations where you are dividing by a single-digit number and can make use of your tables facts to help you do this. The implication is that long division occurs in situations outside the range of tables facts (although I'm not sure whether division by 12 would come into this category if children still learned their 12 times table). Under the rubric 'short division' on page 69 (Y456) we have:

$$\begin{array}{r} 6 \overline{)196} \\ -180 \quad 30 \times 6 \\ \hline 16 \\ \cdot 12 \quad 2 \times 6 \\ \hline 4 \end{array}$$

Answer: 32 R 4

Example

If I were being pedantic I would argue that the  $30 \times 6$  should be  $6 \times 30$  (six multiplied by thirty) since we are thinking 'How many 6s are there in 196', but as multiplication is commutative I shall suppress my pedantic tendencies. The year 6 example on the same page is described as a 'long division' and is set out in a similar manner:

$$\begin{array}{r} 36 \overline{)972} \\ -720 \quad 20 \times 36 \\ \hline 252 \\ -252 \quad 7 \times 36 \\ \hline 0 \end{array}$$

Answer: 27

Example 2

On the other hand, in the QCA (1999) document *Teaching Written Calculations* 'long' seems to refer to layout, with the result that the NNS Example would be described as a long

division – presumably because it comes a long way down the page. QCA's own short division example on page 53 is set out as follows covering two lines:

$$\begin{array}{r} 152 \text{ rem } 2 \\ 3 \overline{)458} \end{array}$$

Example 3

I assume that if I were to use my knowledge of my 15 times table to find  $3462 \div 15$  in the following way:

$$\begin{array}{r} 230 \text{ rem } 12 \\ 15 \overline{)34612} \end{array}$$

Example 4

then this would also be classified as a short division.

The National Curriculum (1999) programme of study for number states that pupils should be taught to *use written methods for short multiplication and division by a single digit* (p. 69) and the attainment target at level 4 includes the word *efficient* before *written methods*. (Incidentally, I have always found it interesting that in official publications produced in this country the most common adjectives used to qualify nouns dealing with calculation methods are *efficient* and *effective*, whereas in Holland they are *fluent* and *flexible*.) It is therefore important that teachers understand what is meant by short division.

Even though my own definition matches that of QCA, I would recommend that teachers follow the NNS interpretation of 'short', and I would do this for two reasons. The first is that the strength of the NNS algorithm, which has come to be known as *chunking*, lies in its differentiability: it allows for different levels of sophistication in children's understanding and confidence in that the less confident can remove small chunks; the more confident can take away larger chunks; and the most confident can subtract the maximum sized chunks. The second reason concerns the compactness of the QCA algorithm. At one level this might be considered to be its strength, but from the perspective of teaching, it is also its weakness as it demands a very good understanding of place value and the principles of exchange. I also believe that it is more compact, dense and non-transparent than the traditional standard algorithm for long division by two-digit numbers which is introduced at a later stage in both the Framework and the QCA document.

Interestingly, HMI (2002) make little mention of division in their report *Teaching of Calculation in Primary Schools*, but they do appear to take a similar view to that of the NNS, identifying short multiplication as  $TU \times U$  and long multiplication as  $TU \times TU$ . This leads me to my first

question which is ‘Why are we bothering to differentiate between short and long division if the layout and calculation procedure for both of them are to be identical in the NNS?’

## W(h)ither Long Division?

The *Framework* (Y123, pp. 68–69), in a section entitled *B: long division*  $HTU \div TU$ , shows the following two calculations adjacent to each other (Examples 5 and 6), with the implication that the first leads naturally to the second. The two look very similar, and, in fact, the only visible difference between the two layouts is the missing zero on the second line of the second example (Example 6).

$$\begin{array}{r} 36 \overline{)972} \\ -720 \quad 20 \times 36 \\ \hline 252 \\ -252 \quad 7 \times 36 \\ \hline 0 \end{array}$$

Answer: 27

Example 5

$$\begin{array}{r} 36 \overline{)972} \\ -72 \quad 20 \times 36 \\ \hline 252 \\ -252 \quad 7 \times 36 \\ \hline 0 \end{array}$$

Answer: 27

Example 6

If you track the build-up to written division in the *Framework* you will find that it is a fairly gradual one [although not gradual enough for Anghileri and Beishuizen (1998)]. It progresses through the following stages: the development of an understanding of the operation, the associated vocabulary and its relationship to subtraction and multiplication; an understanding of the idea of a remainder; the use of closely related known facts, partitioning and the distributive law; and the development of a range of mental division strategies (pp. 54–65). This leads to work on repeated subtraction and culminates in the *chunking* algorithm illustrated in Example 5.

The language and thinking associated with this calculation method runs something like: *I need to find out how many 36s there are in 972. I know that there are ten of them in 360 and so there are twenty of them in 720. If I take this from 972 I get 252. Five 36s are 180 (half of 360), and 180 is 72 less than 252, so seven 36s must be 252. The answer is therefore 20 and 7 which is 27.* However, the procedure shown in Example 6 demands another way of thinking and reasoning: one that utilizes a different vocabulary. I shall assume that I am not expected to teach the algorithm the way I learned it, namely, *36 into 9 doesn't go, so 36 into 97 goes twice. 97 minus 72 is 25. Bring down the 2 ...* The following alternative explanation might provide more indication of the thinking involved in the procedure: *36 into 900 doesn't go (No. I cannot say that, as it does go! Try again). 972 contains 97 tens, so that's two lots of 36 tens with 25 tens left over. Write the 2 above the line in the tens column. If we change these 25 tens to 250 ones and add the 2 we have 252. By trial and improvement we find that 36 into 252 is 7. Write the 7 in the ones column. So the answer is 27.*

I concede that the algorithm shown in Example 6 might make sense to some children, but I have yet to be convinced that there can be a smooth transition from the language and reasoning associated with chunking, which is transparent and supportive of children's thinking, to the language of the standard algorithm which is not. I have argued elsewhere (1999, 2002) that the *quantity* aspect of place value (where 67 is seen as 60 and 7) develops before the more sophisticated *column* aspect (where 67 is seen as 6 in the tens column and

7 in the ones column). The language of chunking clearly involves the quantity aspect, complementing the steps in the procedure, whereas the standard algorithm involves the column aspect as well as a good understanding of the principle of *exchanging*. So my second question is: ‘Is it worth the effort and the potential risk of confusion involved in teaching the algorithm in Example 6 for the sake of a layout which, in this particular case, has one zero fewer?’

## Sharing or Grouping?

When discussing division most writers, including Williams and Shuard (1994), Anghileri (1995) and the *NNS Framework* (Y456, p. 54), differentiate between sharing and grouping, often linking the latter to repeated subtraction. For example, p.54 of the *Framework* (Y456) states that as an outcome: *(Pupils should) understand the operation of division either as sharing equally or as grouping (that is, repeated addition) ...* I always point out that in sharing contexts the answer is the same type of unit as those in the question: if you are sharing marbles the answer will be a specific number of marbles. Grouping contexts give you a different type of unit: if the problem concerns, say, the transportation of pupils, the question will provide you with the number of children and the answer will be in terms of the number of buses.


Williams and Shuard (1994) argue that: *Grouping is also the method usually chosen for the division of large numbers. The division  $324 \div 9$  is usually worked by finding how many nines there are in 324 (grouping) rather than dividing 324 into 9 equal parts (sharing).* It would seem sensible to familiarize pupils with chunking by setting problems in a grouping context, as the use of repeated subtraction allows the removal of groups of differing sizes. I appreciate that one can use repeated subtraction to solve sharing problems by thinking in terms of the sharing of larger chunks, but feel that this demands a more sophisticated level of thinking. Consequently, I would want to teach this aspect later as a generalization of the procedure. QCA (1999) obviously disagree with me, as three of the four examples given for division using chunking with larger numbers are presented in sharing contexts. My third question is: ‘If we are solving sharing problems using chunking (repeated subtraction) how does this sit with the common explanation that division can be interpreted either as sharing equally or grouping with only the latter category associated with repeated subtraction?’

## Inverses?

It is obviously important that children learn about and are able to make use of the fact that multiplication and division are inverse operations. QCA reiterate this point in several of the annual Standards Reports on the national curriculum assessments for 11-year-olds. For example, the 1998 Key Stage 2 report states that: *Children need to appreciate that division is the inverse operation of multiplication.* The National Numeracy Strategy's 5-day course, *Developing Mathematics in Years 4, 5 and 6*, discusses a range of question types that can be solved by inverse operations (Notes for course tutors, p. 99). One of these types is exemplified as *Calculations involving inverse operations:  $950 \div \square = 47.5$ .* I would argue that it is only if you realize or know that 95 is twice 47.5 and that 950 is 95 multiplied by 10 that you can use multiplication to solve the problem. I challenge the reader to use multiplication to find  $949 \div \square = 47.5$ , with or without a calculator. I cannot see how to do this any other way than by dividing 949 by 47.5 (or using ‘guess and check’ methods). This suggests that the inverse of ‘division by’ is actually ‘division by’. So my fourth question is: ‘Does this mean

I am right in arguing that the inverse of division is not necessarily multiplication?

Askew (1998) links multiplication and division when describing five different models: equal groupings; allocating/sharing; arrays; jumps on number lines and scalings. In the equal groupings model multiplication arises when the number of sets is given together with the number of objects in each set. Division arises when the size of each group is given along with the total number of items. Askew argues that *one particular strength of the equal groupings model is the clear link between multiplication and division as inverse operations*. However, given four sets of three objects I would want to think of the calculation as 3 objects multiplied by 4, giving 12 objects. As the inverse of 'multiply by 4' is 'divide by 4', I would then interpret the inverse as 12 objects divided by 4, giving 3 objects. My fifth and final question is: 'This is a sharing context since the answer is the number of objects, so does this mean that the inverse of a 'grouping multiplication' problem is a 'sharing division' problem?

Given my obvious confusion about division I think I shall limit myself to researching multiplication as it is much more straightforward. Or is it? 

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