

Types of Error and Checking Strategies in Calculator Work

by Ian Thompson, Avondale Comprehensive School, Stockport

Estimating or Checking?

Girling¹ differentiates between pre-estimation, where one is dealing with ideas expressed in terms like “The answer should lie between . . .”, and post-estimation which involves statements like “The answer seems reasonable because . . .”. Before looking at calculator errors (or more specifically errors made by calculator users) and at checks which might pinpoint these errors, an attempt will be made to differentiate between estimation and checking in order to avoid Girling’s rather clumsy terminology.

The *Oxford English Dictionary* defines the noun “estimate” in a non-mathematical context as “the price quoted by a contractor for specified work”, and in a mathematical context as “approximate judgement of number”. The former definition suggests that from a temporal point of view the act of estimation should occur before the actual event whose outcome one is estimating actually takes place. If asked to “estimate the answer” or “give an estimation of 59×673 ” it is doubtful whether anyone would work the calculation out first before giving some sort of rough answer. For the purposes of this article “estimation” and “estimating” will signify pre-calculation activity.

“Checking” on the other hand has a different temporal relationship to the calculation under discussion. You would only be asked to check a calculation if it had already been worked out, or if you were expected to do the calculation first and then check it afterwards. Consequently “checking” will be considered a post-calculation activity, and this analysis gives us the simple model shown in Figure 1.

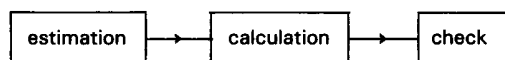


Fig. 1 Temporal relationship between estimation and check.

Estimation involves the obtaining of an approximate answer to a calculation, and it may well be that having estimated the answer you may decide not to do the actual calculation but to use the approximate answer instead. When an approximation is made prior to the actual calculation as a means of ascertaining the probability that the calculated answer is correct, or to guard against the making of errors in the ensuing calculation, then both estimating and checking are being used, in that pre-calculation and post-calculation activities are involved. It seems a reasonable assertion that all the skills and techniques which can be used for estimating an answer can also be used for checking, whereas there are some checking techniques — those which

make use of the answer — which cannot be used for estimation purposes, where estimation has the meaning discussed above.

For the purpose of this article those approximating methods used for estimating answers are classed as rough checking techniques, i.e. for using after the answer has been calculated, although it may transpire that these techniques are best used prior to calculation, as estimation skills, because the presence of the actual answer may well adversely influence the calculating of the rough check.

Analysis of Types of Error

So what does one mean by a check in a calculator context? As Johnson² asserts, when discussing the use of calculators in schools for checking answers which have been worked out using pencil and paper algorithms, what is the point of using the calculator purely as a check when it is usually the best device for performing the calculation in the first place. To suggest the reverse procedure of checking calculator work by using long pencil and paper methods seems a little perverse.

Before drawing up a specific list of possible calculator errors it is proposed to take a more general look at the types of error which may be made in calculator computations. Donaldson³ looked at the mistakes children made doing standard IQ tests, and classified their errors into three types — arbitrary, structural and executive. Arbitrary errors involved some sort of lack of loyalty to what was given in the question, structural errors involved some failure to appreciate the relationships involved in the problem, and executive errors involved a failure to actually carry out the manipulations required, possibly caused by some defect of concentration, attention, loss of hold or short-term memory. Arbitrary errors in a mathematical context, such as ignoring part of the information whilst acting on the rest, and structural errors such as selecting the wrong operation, algorithm or formula, whilst not being specifically calculator errors in that they will have been committed before recourse to the calculator is made, are nonetheless important types of error which need to be taken into account when general checking strategies are being developed in children. The “Does the answer make sense?” or “Is the answer reasonable?” type of commonsense check is a very important general checking technique which is no doubt very difficult to teach.

In the case of the two previous types of error, arbitrary and structural, incorrect or insufficient information is being “deliberately” fed into the calculator. The following calculator errors

which are to be discussed are concerned with the accidental entry of incorrect information into the calculator. These calculator errors are generally going to be classified in the executive error category.

A major difference between pencil and paper calculations and those involving a calculator concerns the use of an extra step — a step one could call “transcribing”. In most cases the calculation to be worked out will be written down either in the form of a standard algorithm or as part of a problem. Even if it arises as part of a real-life situation such as measuring a room for a carpet the most sensible thing to do would be to write the relevant numbers down before feeding them into the calculator, otherwise a room of area $1\,442.6\text{ m}^2$ (instead of 144.26 m^2) might prove expensive to carpet, or expensive in terms of the time needed to re-measure the room in order to check the calculation.

All written calculations have to be fed into the calculator, and it is this process of transcribing the information which is likely to provide a major source of calculator error, for the number to be operated on has to be read correctly, remembered correctly, and entered into the calculator correctly. This process can be compared with making a telephone call after having looked up the number in the directory. The prevalence of “wrong numbers” suggests that many similar errors are likely to be made in a calculator context. The possible types of executive calculator error listed below are described in terms of the end result of the error. No mention is made of whether the error is the result of mis-reading, mis-remembering, or mis-keying.

Some Possible Types of Calculator Error

1. Omitting a digit in a number, e.g. 74 351 instead of 744 351.
2. Entering the same digit twice, e.g. 633 142 instead of 63 142.
3. Omitting one of a sequence of numbers to be operated on, e.g. $27 + 31 + 42 + 53$ instead of $27 + 31 + 42 + 82 + 53$.
4. Entering the same number twice, e.g. $51.6 + 32.4 + 27.1 + 27.1$ instead of $51.6 + 32.4 + 27.1$.
5. A variation on the last two items is losing one's place in adding a column of figures, especially in a list containing numbers which appear several times, resulting in the omission or duplication of a group of numbers.
6. Entering an incorrect digit, e.g. 7 241 instead of 7 341.
7. Entering digits in the wrong order, e.g. 1 234 instead of 1 324.
8. Entering operations in the wrong order, e.g. $3 + 2 - 4$ instead of $3 - 2 + 4$.
9. Entering the wrong operation, e.g. 73×84 instead of $73 + 84$.
10. Omitting an operation, e.g. $32 + 2\ 347 + 85$ instead of $32 + 23 + 47 + 85$.
11. Putting the decimal point in the wrong place, e.g. 51.38×42.64 instead of 51.38×4.264 .

All of these errors could be made at any of the three stages in the transcribing process outlined above although 5 is more specifically connected with mis-reading errors; 7 more likely to be a mis-remembering error; and 10 more probably a mis-keying error. In connection with the seventh error above, Thyne⁴, who looked at the pattern of errors in several thousand additions of single-digit numbers produced by pupils who had completed a batch of tests for the Scottish Council for Research in Education, mentions the frequent occurrence of reversals of

the type $\frac{7}{+6}$. This could be an executive error whereby the

child meant to write 13 but reversed the digits and wrote 31, but other explanations are possible. Donaldson³ comments on the frequency of “reversal of pairs” as a type of error prevalent in answers to series questions in standard IQ tests.

A further possible source of error is incorrect use of the calculator. Different calculators use different types of logic and have different rules for the feeding in of data. Errors will occur when a child who knows the correct mathematical algorithm fails to translate this into the correct calculator algorithm, or when he uses a calculator algorithm which is correct on one model on a model which requires the data to be entered differently. For example, given the problem “Find 5% of 40” a child may correctly reason that finding a percentage involves multiplication, and so what he has to find is $5\% \times 40$. If he then makes the reasonable decision to key $\boxed{5} \boxed{\%} \boxed{\times} \boxed{4} \boxed{0} \boxed{=}$ into a Casio HL-801 he will get the answer 200. If he were to feed the same information into a Rockwell 30R the display would show zero.

Methods of Checking

In this section checks which might pinpoint the errors dealt with in the last section or at least indicate some discrepancy in the original working will be considered. It is important to bear in mind that no check can tell you that an error *does* exist, for the check itself may contain an error or may fail to be sufficiently sensitive. Checking techniques for calculator usage would appear to fall into two distinct categories — rough checks and exact checks — and the type of check that you use depends on factors such as the seriousness of concern that the answer be accurate, and the confidence that you have in your ability to correctly execute the calculation.

Many problems in everyday life have no exact answer. Often it is only necessary to find an approximate answer. For example the question “How many tiles do I need to tile the bathroom?” does have an exact answer, but what is required in answering it is an approximation — preferably an over-estimation to account for breakages.

Rough checks give an approximation to the answer indicating only whether it has the correct order of magnitude or appears reasonable. This type of check involves either an intuitive feeling that something does or does not appear to be in accordance with what you expect, or it involves the conscious or subconscious deployment of some general rounding techniques combined with subsequent manipulation of the rounded numbers.

A rough check correctly carried out can sometimes tell you that an incorrect answer definitely is incorrect, as would probably be the case with errors classified as types 1, 2 and 11 above (provided the operations involved were multiplication, division, raising to a power or finding a root), and also with type 8, 9 or 10 errors. Sometimes the rough check can only suggest the possibility of an error, as in the case of 5 above, when only a few numbers have been omitted or duplicated, or in 1, 2 and 11 where the operation involved is addition or subtraction. However, a rough check is of no use in signalling the existence of an error of type 6 or 7. This is also true for 3 and 4 if more than a few numbers are involved. The problem is that a rough check, however correctly carried out, can never tell you that your answer is correct, all it can do is provide confidence in the accuracy of the answer. Nor can it provide an alternative “candidate” answer whenever it indicates that the original working is incorrect.

Exact checks on the other hand, when correctly carried out, can indicate that an answer is correct if indeed it is; can tell you that an answer is incorrect when it is incorrect; and, dependent on the type of check used, can provide an alternative “candidate” answer. Exact checks, excluding the check which is a direct repetition of the original working, involve operating on the data by transforming the problem into an equivalent one, and it is useful to subdivide exact checks into those which incorporate the answer in the transformation of the problem, where the main techniques used involve inverse operations,

and those which just use the data without the answer, where the main techniques involve the use of the commutative and associative properties (see Fig. 2).

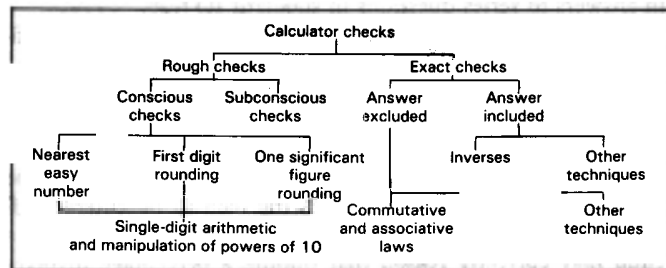


Fig. 2 Classification of calculator checks.

Transformations involving the answers have the advantage that different numbers are fed into the calculator so that errors of types 1, 2, 3, 4, 6 and 7 (especially the last two), which may occur because of the specific properties of the numbers in question, e.g. 1 221 entered as 1 212 or 56 788 entered as 56 789, stand less chance of being made. A disadvantage of this type of check is that the transformation to the equivalent problem may be incorrectly done. A second disadvantage is that the check does not produce an alternative "candidate" answer, which, if different from the original, is available for consideration as being the correct answer, and also available to be checked if the discrepancy between the two answers cannot be readily resolved. A simple example, for which a calculator would not need to be used, is given in order to illustrate this point. The division $20 \div 4 = 4$ can be checked by using the fact that multiplication and division are inverses and so 4×4 should give us 20. When the answer is found to be 16 all that is known is that either the original or the check is incorrect and so the original has to be checked some other way or, more probably, reworked much more carefully.

On the other hand transformations which do not involve the answer do produce such a "candidate" answer. For example, to check $99 \times 7 = 683$ you could work out $(100 \times 7) - 7$, and on getting the answer 693, can either accept this as more likely to be correct, or re-check the original multiplication or check the second calculation. However the majority of checks which do not use the answer, apart from those which are specific to the numbers involved in the question as in the example above, make use of the commutative or associative laws or a combination of both. This type of check has the disadvantage that exactly the same numbers are being entered into the calculator, albeit in a different order, thus giving rise to the possibility of making the same mistake. Other transformations such as $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ have the weakness mentioned above whereby an error may be made in the transformation process.

Commutative/associative checks would signal errors of type 1, 2, 3, 4, 6, 7, 9 and 11, provided that the operations involved possessed these properties, whereas inverse checks would alert us to errors of type 1, 2, 6, 7, 9 and 11. Questions of the type mentioned in 5 would probably best be checked by ticking off the number as it was entered into the calculator, or by moving a card down the column of figures. The calculation of subtotals would help pinpoint the actual error more easily. Errors due to incorrect calculator use are likely to be lessened by the acquisition of a thorough understanding of the idiosyncracies of the particular calculator being used, in addition to an understanding of the general principles of calculator usage.

References

- Girling, M. S. (1976) "Basic numeracy in the electronic age" in *Calculators Have Come*, Mathematical Association.
- Johnson, D. (1978) "Calculators: abuses and uses", *Mathematics Teaching* 85.
- Donaldson, M. (1963) *A Study of Children's Thinking*, Tavistock, London.
- Thyne, J. M. (1954) *Patterns of Error in the Addition of Number Facts*, U.L.P. London.

NEW TEXTS IN MATHEMATICS FROM PERGAMON

NEW TERTIARY MATHEMATICS

C Plumpton, University of London and P S W Macilwaine, Cirencester, England

The series **NEW TERTIARY MATHEMATICS** represents an entirely new approach to the requirements of mathematical students beyond the secondary stage.

A compromise between the modern and traditional treatments has been adopted and the four books complete an integrated course in Pure and Applied Mathematics covering university and professional requirements.

Many exercises and answers are included. Each book is self-contained. SI units are used throughout.

PURE MATHEMATICS: THE CORE Volume 1, Part 1

The material in this book has been carefully selected to include all the topics forming the 'CORE' syllabus as constituting the minimum requirements in mathematics for a wide range of degree courses.

416pp 250 x 172mm 152 illus
0 08 025031 9 Hardcover £17.40
0 08 021643 9 Flexicover £ 6.90

BASIC APPLIED MATHEMATICS Volume 1, Part 2

240pp 250 x 172mm 123 illus
0 08 025035 1 Hardcover £17.40
0 08 021645 5 Flexicover £ 6.90

FURTHER PURE MATHEMATICS

Volume 2, Part 1

416pp 250 x 172mm 105 illus
0 08 025033 5 Hardcover £17.40
0 08 021644 7 Flexicover £ 6.90

FURTHER APPLIED MATHEMATICS

Volume 2, Part 2

210pp 250 x 172mm 89 illus
0 08 025037 8 Hardcover £17.40
0 08 025026 2 Flexicover £ 6.90

Textbook inspection service available for teachers and lecturers.
Please apply to your nearest Pergamon office for details.

All prices are subject to change without notice.
Sterling prices apply to UK and Eire customers only.



PERGAMON PRESS

Headington Hill Hall, Oxford OX3 0BW, UK
Fairview Park, Elmsford, NY 10523, USA