The Role of 'Partitioning' in Mental Calculation Strategies

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The National Numeracy Strategy Framework (DfEE, 1999) contains details of a wide range of mental strategies for single- and two-digit calculations that children are to be taught: counting-on, near doubles, bridging through 10, partitioning, complementary addition, compensation, etc. However, neither this document nor the related NNS training materials attempt to address the issue of the relative importance or usefulness of these different strategies.

In this article I shall focus on one such procedure and attempt to illustrate, with reference to the *Framework,* the way in which it can be developed and extended for use in more sophisticated contexts and with increasingly more difficult calculations. This account will include consideration of mental calculation strategies for each of the four basic operations and of informal written methods for addition and multiplication The strategy selected for scrutiny is 'partitioning' – probably one of the most important mental calculation procedures that need to be developed throughout the primary school. By 'partitioning' I mean the splitting of a given number into several parts (usually, but not necessarily, two) which when added together sum to that number.

In this country we have traditionally over-concentrated on teaching young children to do 'sums', like working out 3 + 4 or 7 - 2. Commercial schemes, old or new, still give plenty of practice in finding such sums and differences – although in the newer schemes the work is usually of the oral and mental variety. However, in some European countries they place more emphasis on the partitioning of numbers: what in this country we would call knowing the 'story' of a number. The assumption is that if a child knows that 7 can be split into 3 and 4 or 5 and 2, then both of the calculations mentioned above are easy.

The reason for the European focus on this aspect of young children's number work is that they believe that the ability to partition will be extremely useful later in the child's development of mental calculation methods. This belief can be seen to be particularly true for strategies that employ 'bridging': a procedure which involves crossing a multiple of ten as an intermediary step in a calculation. For example, a child might calculate 17 + 8 by finding 17 + 3 = 20 and then 20 + 5 = 25, or work out 24-6 by first partitioning 6 into 4 and 2, subtracting the 4 to give 20 and then taking away the remaining 2. The transcripts of Mark and Tim illustrate the bridging strategy for addition and subtraction calculations involving numbers to twenty. Mark, finding 7 + 8, explains:

Well... I had 8 and I knew that 8 and 2 was 10... and then because 7 is an odd number it's got 5 more... and 10 and 5 makes 15.

Tim's answer to 15   9 was simply.

I took away five from 15 and then I took away four... that's six.

The *Framework* introduces partitioning at Reception level, where one of the stated outcomes for children is to be able to "say how up to 10 objects can be separated into two groups". In Year 1 children are expected to be able to partition the larger single-digit numbers into '5 and a bit', and use this skill as part of a mental calculation strategy similar to that used by Lucy (age 7), who, when finding 6 +7, said:

13... Because 5 and 5 and I just added 3 on... I took five out of the six and five out of the seven and I was left with 3.

In Year 2 children are introduced to splitting 'teen' into '10 + something' and in Year 3 they work on partitioning 2-digit numbers into decades and ones (34 as 30 and 4). It is anticipated that increased confidence in their ability to do this will lead naturally to children mentally calculating, say, 34 + 25 as

30 + 20 = 50; 4 + 5 = 9; 50 + 9 = 59.

Revision of work on adding 10 to a multiple of 10 will then help them find sums like 37 + 28, which might be done as:

30 + 20 = 50; 7 + 8 = 15; 50 + 10 = 60; 60 + 5 = 65

Further work on mentally adding 'teens' to multiples of 10 could then lead to an abbreviation of this strategy to:

30 + 20 = 50; 7 + 8 = 15; 50 + 15 = 65.

This procedure is known in the research literature on two-digit addition and subtraction as the partitioning or split method (see Thompson and Smith, 1999[1]). Thompson (1999) quotes Scott (age 7) who explains his answer to 27 + 28 in the following way:

> *Two 20s is 40...seven and eight... take three off eight which would be 10... and three took off eight would be five... so the answer would be 55. (p.149).*

As well as using partitioning when splitting the two 2-digit numbers Scott has also used it for adding the seven and the eight by 'bridging'.

This method can also be used for subtraction, and there are no problems with calculations which do not require regrouping. For example, Surel calculated 68 - 32 in the following way:

> *60 take away 30 is 30... eight take away two is six... so it would be 36.*

With problems like 86 - 39 some children get round the obvious difficulty (6 - 9 = ?) by 'inventing' an ingenious strategy: what I call the 'still more to take away' strategy. It can be explained mathematically in terms of negative numbers, although this is definitely not how children see the method. Research needs to be done to ascertain whether it is feasible to try to teach this method to those children who do not invent it. Abigail's explanation of her strategy for 86 - 39 was:

> *47... I said 80 - 30 is 50...and then you take 3 from 9 is 6... and take the 6 away and take the 3 away.*

Thompson and Smith (1999)[1] describe several other two-digit strategies for addition and subtraction.

In Year 4 children's mental multiplication and division strategies are extended so that children can calculate with two-digit numbers. To achieve this the *Framework* combines those skills introduced in Year 3 for these operations with the doubling and halving skills introduced in Year 1. So, to double 34, children are expected to partition the number into 30 and 4, double the 30, double the 4 and then add the two answers together. Multiplying by four would involve doubling twice, and halving 34 would be done by halving the 30 and adding this to half of the 4. This method is extended in subsequent years to the multiplication of two-digit numbers by larger single-digit numbers, so that by Year 6 children are expected to be able to mentally calculate 86 x 7 by adding 80 x 7 to 6 x 7. This, of course, is just a generalised extension of the doubling method, and makes use of the distributive property:

$a \times (b+c) = axb + axc.$

The section of the *Framework* that covers pencil and paper procedures for the four operations divides them into 'Informal written methods' and 'Standard written methods' (I would prefer 'formal' rather than 'standard' - but that is another story!). Within the 'informal' category we find various examples of the use of partitioning. The recommended algorithm for addition is given the descriptor 'Adding the most significant digits first', and an example given for Year 4 is:

625	
+ 48	
600	
60	20 + 40
13	5 + 8
673	

This algorithm is an almost exact model of the 'split' method described above for mental calculation with two-digit numbers: the digits are treated as quantities (20 is added to 40), and they are added from the left according to their significance, giving a useful first approximation to the answer.

For multiplication the *Framework* recommends the informal 'grid (or 'area') method' illustrated below (see Thompson, 1989 and 1996). The example given for Year 4 children is:

A grid method (TU x U)

For example, 23 x 8 is approximately 20 x 10 = 200

x	20	3	
8	160	24	= 184

A slightly more formal written version of the 'grid method' is included under the 'Standard written methods' heading:

23	
x 8	
160	20 x 8
24	3 x 8
184	

This procedure clearly involves partitioning: the 23 is split into 20 and 3 and each part is then multiplied separately by eight before being recombined to give the answer.

The recent guidance on the teaching of mental calculation strategies (QCA, 1999) also allocates a major role to partitioning, but it confuses the issue somewhat by including five separate categories of partitioning strategy for addition and subtraction, (one of which, 'compensation', does not actually involve partitioning), whilst at the same time ignoring its role in multiplication.

Conclusion

This article has attempted, from a 'connectionist' perspective, to help teachers make more sense of the wide range of mental calculation strategies listed in the *NNS Framework* by focusing on just one important strategy which underpins many of the others. Because of the important role it plays in mental calculation teachers need to ensure that their pupils develop a thorough understanding of the myriad uses of partitioning.

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References

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[1] A free copy of the Nuffield-funded report '*Mental Calculation Strategies for the Addition and Subtraction of 2-digit Numbers*' can be obtained from Ian Thompson, Department of Education, St Thomas St, Newcastle upon Tyne, NE1 7RU.

Calculating Continued...

Racing to one

A game for several players

- **Choose a starting number, for example 31**
- **and a working number from 1 to 9 for use throughout the game, for example 3**
- **Each player can then use +, -, X or ÷ for each turn with the working number for example +3, -3, X3 or ÷3.**
- **Write down all your turns**
- **The winner is the player who can display 1 in the fewest moves.**