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The Role of Counting in the Idiosyncratic Mental Calculation Algorithms of Young Children.

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SUMMARY *Fifty-nine children from Year 2 (6 - 7 year olds) and forty-four children from Year 3 in three different schools were interviewed whilst attempting mental arithmetic calculations commensurate with their age and ability. The children's solutions to a series of simple additions, subtractions and multiplications, and their explanations of the methods used were tape-recorded and later transcribed. In addition to confirming earlier research into the place of counting in the very early years this study suggests that as children progress through school they continue to use counting as an important part of their problem solving repertoire, combining these counting skills in idiosyncratic ways with other learned skills and acquired knowledge. The importance of counting as a basic building block of numerical understanding suggests that teachers of young children may need to place greater emphasis on the development and integration of counting skills during the first few years of children's schooling.*

RESUMÉ *Cinquante-neuf éléves de seconde année (6-7 ans) et quarante-quatre éléves de troisième année, représentant trois écoles primaires, ont été soumis à des questions lors d'exercices de calcul mental adaptés à leur âge et leur niveau. Les réponses proposées par les enfants à une série d'additions, soustractions et multiplications simples, ainsi que les explications concernant les méthodes utilisées pour les résoudre ont été enregistrées, puis retranscrites. L'étude, tout en confirmant la place du calcul, déjà établie par des recherches précédentes, au cours des premières années de scolarité, démontre également l'importance continue du calcul pour la résolution des problèmes tout au long des années scolaires, les enfants mettant leurs connaissances arithmétiques au service d'autres connaissances acquises. L'importance du calcul en tant que base fondamentale de la comprehension numérique entraîne l'hypothèse suivante: les enseignants du primaire doivent porter davantage l'accent sur l'acquisition des connaissance arithmétiques au cours des premières années scolaires.*

ZUSAMMENFASSUNG *Neunundfünfizig Kinder aus dem 2. Schuljahr (6- bis 7-Jährige) und vierundvierzig Kinder aus dem 3. Schuljahr wurden bei Kopfrechnungsaufgaben befragt. Die Lösungen einer Reihe von einfachen Additions-, Subtraktions- und Multiplikationsaufgaben durch die Kinder sowie ihre Erklärungen der angewendeten Methoden wurden auf Band aufgenommen und später übertragen. Neben der Bestätigung früherer Untersuchungen über die Bedeutung des Zählens in den frühen Lebensjahren lässt diese Studie darauf schliessen, dass Kinder in ihrer Entwicklung in den Schuljahren das Zählen weiter als einen wichtigen Teil ihres Problemlösungs-Repertoires benutzen, indem sie die Kompetenzen im Zählen auf* *charakteristische Weisen mit anderen erlernten Kompetenzen und erworbenem Wissen verbinden. Die Wichtigkeit des Zählens als Grundbaustein des numerischen Verständnisses gibt dem Gedanken Nahrung, dass Lehrer für die jüngeren Kinder der Entwicklung und Integrierung von Zählkompetenzen in den ersten Jahren des Schulunterrichts wahrscheinlich grössere Bedeutung beimessen sollten.*

RESUMEN Se entrevistó a cincuenta y nueve niños del Curso 2 (de 6 a 7 años) y cuarenta y cuatro niños del Curso 3 de tres escuelas diferentes ofreciéndoles cálculos aritméticos mentales adecuados a su edad y habilidad. Las soluciones que dierou a una serie de sumas, restas y multiplicaciones sencillas y la explicación de los métodos que habían seguido se grabaron en cinta para transcribirse posteriormente. Además de confirmar previas investigaciones sobre la utilización del contar en los primeros años, este estudio sugiere que a pesar del desarrollo, continúan usando el contar como parte importante de su repertorio matemático, combinándolo de forma idiosincrásica con otras habilidades y conocimientos adquiridos. La importancia del contar como un bloque sobre el cual construir la comprensión numérica sugiere que los profesores de niños pequeños han de poner un mayor énfasis en el desarrollo e integración de las habilidades que supone el contar durante los primeros años de escolaridad de los niños.

Keywords: Algorithm; Counting; Derived facts; Calculation strategies.

Introduction

Gelman and Gallistel (1978) have argued that counting is governed by implicit knowledge of five basic principles:

> one-to-one correspondence - each item to be counted is labelled with one, and only one, number name;
> stable order - the labels assigned to the objects to be counted are ordered in the same sequence across trials;
> cardinality - the last number name assigned in a count represents a property of the entire set - its numerosity or cardinality;
> abstraction - objects of any kind may be counted;
> order irrelevance - objects may be counted in any order provided no other counting principle is violated.

Two different views have been put forward concerning the emergence of these principles in early childhood. Gelman and Gallistel (1978) suggest a *principles first* theory, claiming that these principles are innate and guide the acquisition of counting procedures. Others (Fuson and Hall, 1983; Fuson, 1988) argue for a *principles after* theory whereby the counting principles are considered to be progressively abstracted after repeated practice of rote counting in a variety of social situations. Meck and Church (1983) suggest that some animals possess a 'preverbal counting mechanism', and, building on this work, Gallistel and Gelman (1992) have argued that human infants are equipped with a similar preverbal counting mechanism, and that it is this mechanism which provides the source of the implicit principles which guide the acquisition of verbal counting.

There exists a wealth of research evidence to suggest that children starting school possess a much richer experience and more sophisticated understanding of number than was previously thought (Fuson, 1988; Hiebert, 1984; Hebbeler, 1977). Gelman and Gallistel (1978) suggest that even children who fail the standard Piagetian number conservation tasks can often operate successfully with small numbers, and

Ginsburg (1977) draws a comparison between the natural informal mathematics that young children bring with them to school and the formal algorithmic mathematics that they encounter once there. Aubrey (1993) details the level of understanding of a group of children (average age 4 years and 6 months) on ten separate aspects of number knowledge. She shows that many of these children, who have been in school for just two weeks, have an unexpectedly good intuitive understanding - without possessing the formal conventions for representing them - of many of the number concepts that the school envisages teaching them over the next few years.

Carpenter and Moser (1983) have identified the following levels of addition strategies used by young children when solving simple word problems:

- *counting all* - where a child solving a simple addition problem such as 2 + 3 first counts out two blocks followed by three other blocks, and then finds the total by counting the number of blocks altogether;

- *counting on from the first number* - where a child, finding 2 + 3, begins the count by repeating the first number and then continues the count by starting from that number. For example, a child might say:
  "Two..... three, four, five. There are five";

  counting on from the larger number - where a child proceeds as in the previous example, but begins the count from three, reasoning that starting from the larger number will mean that less counting will be involved;

- *using known number facts* - where children give immediate responses to those number bonds which they know by heart - usually the simpler number bonds such as the smaller doubles like 2 + 2 and 3 + 3;

- *using derived number facts* - where children use a number bond that they know by heart to calculate one that they do not know. In the initial stages there is a tendency to use the doubles, so that 6 + 5 might be found by saying:
  "Five and five is ten and one more makes eleven", or
  "Six and six is twelve, but it's one less so it must be eleven".

Steinberg (1985) documents the spontaneous derived-fact strategies of a class of 23 second-grade children, and suggests that tests administered at the end of an instructional unit showed that children's use of derived-fact strategies more than doubled. She argues that derived-fact strategies can actually be taught to young children.

Groen and Resnick (1977) have shown that even preschool children can invent for themselves calculation procedures that they have not previously been taught. A group of five-year olds was taught the 'count-all' strategy, and after several practice sessions many children spontaneously progressed from 'counting-all' to 'counting-on', and some even adapted the latter strategy to 'counting-on-from-larger': a strategy which implies an intuitive understanding of commutativity. In addition to this, Houlihan and Ginsburg (1981) have suggested that first and second graders actually select their counting strategies according to the size and familiarity of the numbers involved.

In order to ascertain the extent to which counting heuristics might be used by children working with the different basic mathematical operations it was decided to interview a group of young children involved in the performance of a range of mathematical calculations in school.

Methodology

As the main purpose of the study involved an attempt to gain access to children's cognitive processes, a pilot project was conducted with a class of Year 1 children (5 to 6 year olds) in order to ascertain the optimum way of collecting the necessary data. As a result of this pilot study it was decided that the cohort to be interviewed should be Year 2 (6 to 7 year olds) and Year 3 children; that they should be interviewed in pairs rather than alone; that the interviews should be recorded on audio tape, and that transcriptions of the tapes should be made.

The sample would be an opportunity sample comprising three schools that the researcher had worked in, and therefore knew and was known by the children. Two of the schools were in middle class rural areas and one was an inner city school with a mainly working class catchment area. By the end of the project information concerning the idiosyncratic mental calculation strategies used for addition, subtraction and multiplication by 103 children had been collected by the author.

A consideration of the literature on interview techniques suggested that, given the ages of the subjects involved, protocol methods based on semi-structured interviews using flexible questioning techniques contingent on the children's responses would be the most appropriate data-gathering instrument. Because it had been observed that some children in the pilot study had managed to forget the numbers they were working on part way through a specific problem, it was decided to have a written version of each calculation on a card which would be placed in front of the child after the interviewer had given the question orally. It was stressed that the interviewer was interested in the **methods** that the children were using, and that it did not matter if the answers were not exactly correct. The children were told that they did not have to use methods that they had been taught in school, but could use their own favourite methods. Field notes were kept, but were confined mainly to a description of any nonverbal behaviour that might cast further light on the strategies that the children were using.

Transcripts of the interviews were studied, and an attempt was made to develop abstract categories that might explain the actions and intentions of the children by subjecting the data to several stages of analysis. All responses were searched for promising lines of enquiry such as points that recurred and basic categories that might best describe the data. As categories began to emerge they were explored by comparing data derived from different children. The technique of 'theoretical sampling' was used whereby initially responses with minimal differences were compared to bring out the basic properties of a category. This was then followed by an attempt to maximise the differences between the responses compared in order to refine the categories developed still further. Finally, using the technique of 'analytical deduction' a systematic search was made for falsifying evidence which might lead to a modification of the ideas developed. The explanatory validity of the research was sought by providing access both to the data and to the proposed interpretations to interested colleagues throughout the course of the research.

Results

The classification of calculation heuristics listed below was derived from an analysis of the transcripts. Specific examples of children's responses are used to show how some children, even when they have learned more sophisticated calculation strategies, sometimes combine these heuristics with basic counting. There is no suggestion that all children do this.

Addition

A wide range of additive calculation methods were generated by the children in this study, and examples were found of each of the addition strategies identified by Carpenter and Moser (1983). These heuristics are discussed under the following headings: counting-all, counting-on, doubles, going-through-ten, skip-counting and regrouping. The categories are illustrated by including examples of children's work which involve the use of counting as a part of their overall solution strategy.

Counting-all

Several children were still at the stage of 'counting-all', where they used their fingers to model the numbers to be added. For example, to find 4 + 3 a child would first count out four fingers and then count out three more. The number of raised fingers would then be counted to give the answer to the problem.

Counting-on

Many children used some form of 'counting-on' strategy. To make the transition to this heuristic children have to realise that the count of the first number can be abbreviated by using the cardinal term for this addend to serve as the starting point for the count of the second added - what Fuson and Hall (1983) describe as the 'cardinal-count transition'. This is the reverse of the 'count-cardinal transition' involved in the normal act of counting, where the number name ascribed to the last element counted in a given set is taken to be the cardinal value of that set.

The 'counting-on-from-first' strategy is clearly exemplified in Jacqueline's answer to 5 + 6 which was:

'"5...... 6.. 7.. 8.. 9.. 10.. 11. It's eleven."

On the other hand Steven's explanation of how he found his answer to the same problem illustrates how the drive for 'cognitive economy' - the desire to reduce the load on working memory - obliges him to adopt the 'counting-on-from-larger' strategy whilst at the same time showing that he has some awareness of the commutative property of addition (5 + 6 = 6 + 5):

"Well, I took the big number first.... I said six in my head and counted five more..."

The utilisation of commutativity becomes a more useful labour-saving device as the difference between the two addends increases in size. One or two children extended 'counting-on-using-fingers' by involving imaginary appendages. Emma's response to 11 +12 was

"23....... I had 12 and I added 10 on and a 1... I didn't have enough fingers...... I just said one more".

This 'finger-extension' strategy will be looked at again in the section dealing with subtraction.

Doubles

This was the most common 'derived-fact' strategy used by the children, and is clearly illustrated in Hannah's answer to 5+6:

"I looked back in my memory... six and six is twelve, so it's one less".

Some children had favourite doubles which they used in a variety of situations, whilst others used a doubles fact and then used counting-on. Ben (finding 5+7) said:

"10... 11, 12. I counted in my head".

He actually counted aloud, but when pressed it transpired that he meant that he had counted two objects in his head. It is probably the case that other children use this calculating strategy but the fact that they perform the count silently in their heads means that no one else becomes aware of this.

Other responses involved a 'doubles-plus-or-minus-one' or a 'doubles-plus-or-minus-two' approach. Jane was seen to put up three fingers whilst working out 7 + 4, and ,when asked how she arrived at her correct answer, replied:

> *"Four and four is eight... and then if you put a seven instead of the four it's three more... so it's three more than eight".*

This extension of the basic 'doubles-plus-or-minus-one' to a 'doubles-plus-three' strategy was much more rare!

Going-through-ten

The mathematical idea underlying this strategy is 'complements in ten'. For example, 7 is the complement in ten of 3, and 1 is the complement of 9. This idea is covered in most mathematical schemes by the 'story of ten' type of activity, but very few of these schemes offer more than one or two such activities. The concept is not usually treated as the important building block for mental arithmetic that it would appear to be. Vicki worked out 8 + 5 by, in her own words,

> *"...taking the two off it and putting it there".*

Clearly she had decided to make the eight into a ten by adding the two that she had removed from the five. The subsequent addition of the ten and the three was then more easy for her to do. Paul's explanation of his answer to the same problem was:

> *"I made the eight into ten and went 11, 12, 13".*

In order to use this strategy effectively children need to be able to do the following: ascertain what is needed to build one of the numbers up to ten; partition the other number into two appropriate parts, and then add these two parts separately by counting-on or by making use of their knowledge of the effect of adding a single digit number on to ten. The 'complements-in-ten' strategy, with or without counting, was used much more frequently with subtraction than with addition, and is considered below in more detail.

Skip-counting

Counting in multiples of two, three or indeed any number is what is meant by 'skip counting'. Children often learn this skill, and some teachers teach it, as a preparatory activity for the learning of 'tables facts'. Ben's answer to the calculation 4 + 5 was:

> *" 4...6...8...9".*

It was difficult to ascertain exactly why Ben had tackled the problem in this way, but it related to his visualising the five in standard 'domino' formation, then adding the two pairs of dots by counting on in twos from four, and finally adding on the remaining dot. He later used a similar strategy when calculating 13 +15:

> *" 33.... I counted in fives after fifteen and added three on".*

Ben had actually counted on one five too many in his original method. The many examples in the data suggest that children rarely make errors when using their own personal heuristics. However, an analysis of the thinking involved in Ben's solution suggests a potential source of error. He first had to recognise that fifteen was an element in his five times table and that thirteen comprised two (or perhaps just 'some') fives and a three. Once he started counting on from fifteen in fives he had also to keep track of the number of fives he was counting. One possible reason for his error was that he was distracted by the fact that the number he had begun counting from - fifteen - contained three fives, and so this made him count on three rather than two fives.

Regrouping

This strategy is similar to the regrouping or decomposition method of subtraction. It involves the breaking down of one or more of the numbers into parts and then proceeding to operate with or on the various parts. The 'going-through-ten' strategy discussed above could be described as a 'regrouping' procedure. However, with larger numbers these parts more often than not comprise the tens and the units. In the research reported here it is of interest that the tens were operated on **before** the units in 95% of cases. A good example is Lucinda's lucid explanation of her method for finding the solution to 35 + 26:

> *"I added three and two up first.... that's fifty.... Five and six is eleven, so I took a ten off and made it to a sixty... and I made it to sixty one".*

Lucinda not only uses regrouping as an overall strategy for the problem but she also uses it in the middle of the calculation in order to deal with the extra ten she has obtained from adding the units.

Alan could not put into words how he had solved 27+28, but his actual answer reveals his thinking quite clearly:

> *"40... 47... 48... 49... 50... 51... 52... 53... 54... 55...".*

He has regrouped in order to deal with the two twenties first of all; has put the seven from the twenty seven onto the forty to give him forty seven, and has then counted on the remaining eight to give him the correct answer. Rachel (25+14) proceeded in a slightly different manner. As she worked out her answer she said:

> "25...35...36...37...38...39".

She has regrouped the fourteen as a ten and a four, has added the ten on to the whole of the other number - rather than just the tens part - and has then counted on the remaining four.

Subtraction

The learning of subtraction number bonds appears to run concurrent with the learning of addition bonds, except that the former knowledge is acquired at a slower rate. Many children use the interconnection between addition and subtraction bonds to solve some subtraction problems. Beth's explanation of her correct answer to 7 - 4 was:

> *"Well, four and three makes seven.... so it's three"*

Children who would not normally use counting when solving addition problems could well still be using such methods for subtraction.

Four main strategies for dealing with subtraction situations were identified. These were: 'counting out' (separating from), 'counting-down-from', 'counting-up-from' and working 'down through ten'. The first two occurred with greater frequency than the latter two. Baroody (1984) discusses a further heuristic which he calls 'counting-down-to', but no examples of this particular strategy were observed in this study. 'Counting-down-to' is the reverse of 'counting-up-from'. In order to find 7 - 3 you count the counting words from seven to three whilst tallying the count. A child solving this problem might say:

> *"7........6.. 5.. 4.. 3........It's four".*

The strategy is more economical for situations where the two numbers are close together or are two-digit numbers, but the procedure does have the potential for error since it is easy to include the number representing the minuend in the count. It is difficult to find a reason for the dearth of examples of the 'counting-down-to' strategy in this study, other than to point out that when the children were dealing with two-digit numbers they used different procedures which generally did not involve counting techniques.

Counting out

This method involves the modelling - usually on the fingers - of the number to be operated upon (the minuend). The required number of fingers is set up and then the number to be taken away (the subtrahend) is removed either by counting the fingers down, or by using prior knowledge of finger totals. The remainder is then dealt with in a similar way. This strategy can involve some children in three separate forward counts, although many youngsters soon learn to set up or remove a given number of fingers quite quickly and then read off from their fingers how many remain. Instead of modelling using fingers some children use a mental representation of the situation. Patrick, working out 7 - 3, said:

> *"Four.... I just knock down skittles in my head".*

Further into the interview, having just given the correct answer to 13 - 6, he explained:

> *"I just had a game of skittles".*

This strategy can be very successful when dealing with numbers of ten or less, but - because of a finger shortage problem - it is less useful with larger numbers. Many children perceive the need for a more sophisticated technique when dealing with numbers greater than ten. Others, however, show surprising ingenuity in modifying a strategy that has brought them success. Anna had been correctly using 'counting-out' to answer some simple subtractions, and was asked to calculate 11 - 6 with a view to helping her to realise the inadequacies of this technique with these particular numbers. Fingers were set up and the correct answer was given. When asked how she had worked out the answer even though she only had ten fingers, she replied:

> *"I counted the newspaper".*

Anna had used my newspaper, which was lying on the table, as the eleventh object in her collection. She proceeded to 'remove' that object first before returning to the familiar territory of her ten fingers to deftly take away the remaining five objects. She later calculated 15 - 9 by imagining five extra objects.

Joanne, however, was the expert in this particular technique. The following explanations for three different subtractions reveal her creative talents and her apparent obsession with her bodily parts:

> *"I used my two legs" (12 - 4.) "I used both strips of my tracksuit as well as my legs. You could use your arms and your legs instead" (14 - 6). "I took that one away (points to one arm)...then that (points to other arm)... and then my head" (13 - 7).*

Counting-down-from

It appears to be the case that when children discover or invent a more powerful or less time-consuming method for solving a particular problem they tend to use it in all situations where they feel confident with the numbers involved. However, they do sometimes 'regress' to less sophisticated methods on occasions where the numbers are unfamiliar or larger than those they are used to. This is to be expected since most adults tend to operate in a similar manner. It also appears to be the case that they sometimes explain their methods to 'confused' researchers by referring to a 'simpler' strategy that they may well have abandoned at an earlier stage in favour of a less time consuming method. 'Counting-down-from' can be used to deal with all those subtractions easily solved by using 'counting-out', but it also has the advantage of being more effective for the solution of problems involving numbers greater than ten. Richard found 7 - 3 by saying:

> *"7....... 6.. 5.. 4."*

whilst putting up three fingers, and Rebecca correctly worked 23 - 9 by counting backwards starting from 22 and tallying the count on nine of her fingers.

This strategy was the most common subtraction procedure used by the children in this sample. However, an analysis shows that it is quite a sophisticated strategy. Children need to be able to execute successfully the following sub-skills: count backwards from a specified number; count backwards a specific number of steps, and employ some suitable 'keeping track' device. In addition, they must also have already perceived a need to keep track of the numbers being counted. Fuson (1988) has provided a detailed breakdown of the various keeping-track methods used by children when counting on and counting back.

Baroody (1984) argues that, because this procedure involves a forward count during the keeping track phase, 'counting-down-from' involves two simultaneous processes which in effect go in opposite directions. This fact could account for the occasional errors that were made by children using this technique. Graham gave the answer "five" when finding 7 - 3. His counting back strategy had let him down because he had counted *"7... 6... 5"* instead of *"6.. 5.. 4"* - an error made by one or two other children in the sample. The backward and forward count and the keeping track method have all been correctly executed. Graham has simply started his count at the wrong place. Some Year 3 children used this strategy in connection with their place-value knowledge to tackle more difficult two-digit subtractions. Daniel found 33 - 18 in the following way:

"It was 33 and I took away the ten.. that was 23...then I took away the eight".

When asked how he had taken away the eight Daniel replied that he had counted back in his head. There were other children who used this extension of the standard strategy.

Counting-up-from

Carpenter and Moser (1984) suggest that children prefer to use the 'counting-up-from' rather than the 'counting-down-from' strategy. This appears to be borne out by their results since only 18% did not use this strategy at all. Baroody (1984), on the other hand, argues that children use the 'counting-down-from' strategy more frequently because it provides a more accurate model of their informal concept of subtraction as 'take away'. The results of this study confirm Baroody's (1984) position, since only **three** of the 103 children interviewed used this procedure. One possible explanation for the use of this strategy by so few children is that they were not given a context within which to do the calculation, but were - in the case of subtraction - asked questions like 'What is ten take away two?' whilst simultaneously being shown a card with 10 - 2 written on it. The verbal format used presents the situation as a 'take away', and perhaps thereby militates against the children's using a 'counting-up-from' strategy.

A consideration of Gillian's answer to 7 - 3 should clarify the way in which this procedure operates:

"3...... 4.. 5.. 6.. 7....It's four".

The final part of her answer came from her reading off the number of fingers she had raised whilst tallying her forward count. Here we have a 'count-and-tally-and-then-count-the-tally' procedure. What is being counted in this case is the number of counting words from three to seven. It is of interest to note that this strategy involves the child in raising the finger that is normally mapped onto the counting word 'one', but, instead, mapping it onto the counting word 'four'. This is quite a subtle adaptation of the basic counting procedure.

Down-through-ten

This strategy was used on many occasions and by a number of different children. It employs aspects of two sub-strategies discussed in the section on addition: 'complements-in-ten' and 'regrouping'. Tim, working out 15 - 9, said:

"I took away five from fifteen and then I took away four... that's six".

Vicki's account of how she calculated 23 - 9 provides a lucid explanation of the thinking involved in the execution of this procedure:

"Three and six make nine... and I took three away to make twenty... and I had the six to make four... so it's fourteen". (Vicki mentions 'four' because it is the complement-in-ten of six).

None of the children in the sample had been taught this strategy, and yet an analysis of the technique suggests that it is very subtle and quite complex. The units digit of the larger number tells you how many to take away first in order to reduce this number to ten or a multiple of ten. You then have to break the smaller number into two parts, one of which is equal to the units digit of the larger number. The other part tells you how much you still have to take away from the ten (or multiple of ten). The answer is the complement-in-ten of this number or a multiple of ten added to this complement. Imagine trying to teach this technique formally!

One child combined this strategy with a counting procedure in order to obtain his answer. Daniel (24 - 7) explained his answer thus:

"I took away four to twenty... then I took away three and I got seventeen".

When asked how he took away the three he said:

"I went 20... 19... 18... 17".

Had more children been asked how they tackled this final stage of the calculation it could well be that other similar examples would have come to light.

Multiplication

Many of the children in this study had had little experience of multiplication. However, there were some children whose grasp of number seemed sufficiently developed to warrant asking them a few basic multiplication questions. Consequently the range of strategies was more restricted, although counting was very much in evidence. For example, to find, say, three sets of four, several children counted out all three sets on their fingers. Melissa's answer to this question was:

"8....9..10..11..12" (doubling with counting on).

Rebecca took this strategy one stage further when she worked out 6 x 6 by first doubling six. She then doubled twelve, and finally counted in ones from 24 to 36. Some children found 4 x 5 by skip counting in fives. Charlotte's response to 4 x 6 used skip counting with counting on:

"6...12...18...19...20...21...22...23...24"

Both of these strategies were used quite often. Kevin gave an interesting answer to 6 x 6 when he said:

"Something like 36... six and six makes twelve...24...36".

Other children used a known fact combined with some form of counting. For example, Camilla used a sophisticated procedure, but made an unfortunate mistake in the process. Her solution to 6 x 9 - a hard one reserved for the more able Year 3 children - went as follows:

"Fifty one....... six tens are sixty and then I counted down nine".

Her error came from counting down 'nine' rather than 'six', but her method of calculation was quite ingenious for someone only 7 years and 8 months old! Tim and Richard were being interviewed together and Richard had just given the answer nine to 2 x 3. After I had told him that he had worked out 3 x 3 instead of 2 x 3 I then asked

Tim to calculate 3 x 4. With a smile on his face he gave the answer;

"Twelve...I added three more from nine".

When asked where the nine had come from he said:

"From Richard".

During the early acquisition of multiplication facts it would appear that children rely heavily on a combination of doubles, skip counting, known facts and counting techniques to help them work out unknown facts.

Discussion

The research provided confirmatory evidence of Carpenter and Moser's (1983) suggested performance levels for addition and subtraction strategies. In the area of derived fact strategies a more widespread use of counting heuristics was observed than had been reported in the literature. The children had learned a wide variety of counting skills which included: counting forwards in ones; counting up from a given number; counting backwards in ones; counting down from a given number, and 'skip counting' - which involves counting in multiples of a chosen number. They selected from within this range of strategies in order to make calculations involving addition, subtraction or multiplication. Many children had also cleverly combined these counting skills with other skills or knowledge such as: memorisation of the doubles; quick calculation of complements in ten; addition of a multiple of ten; subtracting down through ten, and simple multiplicative skills. This they did without having been formally taught these strategies.

Several researchers (Gelman and Gallistel, 1978; Carpenter and Moser, 1984; Fuson, 1983; Groen and Resnick, 1977; Hebbeler, 1977) have demonstrated the important role that counting has in the **initial** acquisition of basic calculation skills and number knowledge by preschool children and those in their first year of schooling. This study suggests that as children progress through school they continue to use counting as an important part of their problem solving repertoire, and that they develop creative adaptations of this basic skill by learning to combine their counting skills with other newly acquired mathematical skills, facts and knowledge. Lankford (1974) found that even children of secondary school age still used counting in the multiplication of whole numbers. In his sample of 176 thirteen year olds, 63 used counting or addition - usually to derive unknown combinations from known ones.

Clements (1983) found that of two classes of four-year olds - one taught classifying and ordering skills and the other taught rational counting strategies - the 'number skills' group outperformed the 'logical operations' group on both the number concepts and the logical operations tests. He concluded that logical operations need not constitute a prerequisite to the learning of number concepts. He argues that the 'logical foundations' approach should perhaps be combined with a 'rational counting strategies' approach for teachers to achieve the best results with early years children.

The observed use of counting by older children in a variety of different situations in this study suggests that counting is a multifaceted skill which comprises a variety of sub-skills, and which also constitutes a calculating sub-skill in its own right, in that it is often combined with existing skills and knowledge to generate other new skills and knowledge. It could well be that through the application of increasingly more efficient counting procedures young children gradually discover or construct for themselves a wide range of number concepts. If this is indeed the case, then perhaps teachers need to give greater emphasis to the development of counting skills throughout the primary school mathematics curriculum than they do at the present time.

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