The influence of structural aspects of the English counting word system on the teaching and learning of place value

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The concept of place value is fundamental to a child's understanding of number and number operations. It is difficult to envisage children successfully engaging in mental or written calculation without a thorough understanding of this basic concept. The importance attached to place value in mathematics education has resulted in a substantial body of research literature concerned with the teaching and learning of this concept, and the topic has been researched from a wide range of different perspectives. Bednarz and Janvier (1988) have focused on the 'grouping' aspect of the concept; Carpenter and Fennema (1990) have investigated a 'problem-solving' approach to place value; Jones *et al.* (1994) have produced a framework to describe children's thinking in this area, and Murray and Olivier (1989) have developed a four-level model of children's conceptual understanding of place value.

In an article which develops an approach to the teaching of place value to children in first and second grade Fuson and Briars (1990) discuss the teaching of operations involving two- and three-digit numbers (multi-digit numbers). They argue that the English spoken system of number words constitutes a 'named-value' system for the values of hundred, thousand and million. In such a system a number word is spoken and then the word immediately following it gives its value. For example, in the number 'four thousand eight hundred', the *thousand* gives the value of the four, so that the reader knows that it is not four tens that are being referred to. In a similar way the *hundred* gives the value of the eight. The authors contrast this with the system of written multi-digit number marks, which they describe as a 'positional base-ten' system wherein the values which were *explicitly* named in the spoken system are now *implicitly* indicated by the relative positions of the number marks. They argue that, in order to understand these two systems, children need to construct named-value and positional base-ten conceptual structures for the words and the marks, and also need to relate these conceptual structures to each other and to the actual words and marks.

It is important to note that the English counting word system has a named-value structure only for values of a hundred or greater: two-digit numbers do not behave in the same way. By contrast, Asian languages retain the named-value structure throughout the entirety of their counting systems. Oral counting in Japanese and Chinese begins as in English by proceeding from *one* to *ten*. However, *ten* is then followed, not by *eleven* but by the equivalent of *ten-one, ten-two, ten-three ... ten-nine, two-ten, two-ten-one,* etc. After *two-ten-nine* comes *three-ten*, and the decade numbers (30, 40, etc.) continue this pattern up to *nine-ten*. Within this system, the number which is one less than a *hundred* is *nine-ten-nine*. This structure can, therefore, be seen to be highly regular, logical and systematic.

Fuson and Briars (1990) argue that the lack of consistency in our system makes it all the more difficult for English-speaking children to construct named-value meanings for two-digit numbers. They suggest that because the English counting word system does not offer pupils the same verbal support as do Asian languages it is all the more important that classroom help is provided to facilitate the construction of appropriate conceptual structures. They recommend a particular approach to teaching addition and subtraction using Dienes base-ten blocks which, they argue, will help children to integrate the different conceptual structures.

The approach described is similar in some respects to that advocated in this country. It involves the important concept of 'exchanging' or 'trading', where, for example, a 'ten stick' may be exchanged for ten single cubes, or vice versa, in order to render a particular calculation easier to execute. There is, however, one aspect of their recommendation that differs substantially from the approach taken in this country. Nearly all British commercial mathematics schemes advocate the use of base-ten equipment for teaching young children to add and subtract two-digit numbers and even for performing single-digit calculations whose sum exceeds ten. Fuson and Briars (1990), on the other hand, suggest that the materials should be used by children only when they can work successfully with number bonds to 18. They also make the interesting recommendation that when multi-digit work is introduced it should be via the addition of four-digit rather than two-digit numbers.

Their argument is that by working with that part of the English spoken system of number words that is completely 'named-value', children will come to appreciate the extent to which operations on the symbols can be modelled by operations on the blocks. They believe that, by initially bypassing two-digit numbers, teachers will avoid subjecting their pupils to the confusion that can develop when working with non-named-value numbers. In order to explore the feasibility of this recommended teaching sequence for place value, it was decided to investigate some of the evidence concerning the strategies that young children use to solve two-digit additions and subtractions mentally.

Methodology

A dataset of young children's performance on a selection of addition and subtraction problems was searched in order to find examples of calculations involving two-digit numbers. The aim was to ascertain the extent to which the children's responses would support Fuson and Briars's (1990) recommended teaching strategy. The dataset involved tape-recorded examples of the mental calculation strategies of 103 children in Year 2 and Year 3, reported in Thompson (1995). The sample comprised children of that age because they were considered less likely to have practised at length the more formal standard written calculation algorithms and more likely to be working at the limits of their ability with two-digit calculations. It was also felt that such children might generate strategies of their own, and that these methods would therefore be more likely to reveal their thinking and their understanding of place value.

Discussion

Thirty-six children in the sample had produced evidence of successful mental calculation with two-digit numbers. Only three had found the answer to 27 + 28 using language equivalent to 'two tens plus two tens', and a further two children had found the sum by working from right to left, adding the units first. All the others had operated with 'twenties', and had worked from left to right. The responses of seven different children to this particular sum are discussed below:

> *Elspeth.* Twenty and twenty is forty seven and eight is fourteen, fifteen ... so that's fifty-five.

This was the most common strategy. It involves partitioning both numbers into the quantities that each digit represents (*not* into 'two tens and seven units') and then adding those quantities separately before recombining them.

> *Scott.* Fifty-five ... Two twenties, as Andrew said, is forty. Seven and eight ... if there's seven, take three off eight, which would be ten ... which would make fifty. Three took off eight would be five ... so fifty-five.

Scott tackles the initial stages of the sum in a similar way to Elspeth, but adds the seven and the eight differently. She used 'near doubles', whereas he uses his knowledge of 'complements in ten' to generate a further ten which he then immediately adds to his initial subtotal.

> *Shane.* Two twenties are forty and seven makes forty-seven and eight makes ... fifty-five.

Shane also partitions the numbers before adding them, but proceeds by using a cumulative method. First of all he adds the seven on to the forty, then he adds the eight on to his running total.

> *Mark.* Fifty-five ... 'cos, you know, I did twenty and twenty is forty . Forty-eight and another two from the seven is fifty, and I've got five left . so fifty-five.

This explanation clearly illustrates the 'bridging through ten' strategy which is based on knowledge of complements in ten. Mark has chosen to deal with the larger of the two units first, probably because adding the eight on to the forty takes him slightly closer to the next multiple of ten. All he then has to do is partition the seven into a two and five; add the two on to the forty-eight to take him to fifty; then add on the remaining five.

> *Jacqueline.* Fifty-five ... I got the two twenties that's forty, then I added the seven and counted on eight.

This method is similar to the previous two, except that Jacqueline seems to lack the confidence of Shane. Instead of calculating eight added to forty-seven in one fell swoop she uses 'counting on' for the final stage of the sum. However, she does appear to have more confidence in her calculating ability than Alan, who counts-on for both of the unit quantities.

> *Alan.* Forty ... forty-seven, forty-eight, forty-nine, fifty, fifty-one, fifty-two, fifty-three, fifty-four, fifty-five.

Alan's ability to partition the twenty-seven and the twenty-eight and to add together the two twenties would suggest that he should also be able to add seven on to a multiple of ten. No doubt in normal circumstances he could do so, but it is quite common for children to 'retreat' to a safer, if less sophisticated, strategy when working with numbers or contexts that, for some reason or other, make them feel a little less confident. The situation is similar, in some respects, to adults making the decision to add a set of numbers on paper rather than in their head, or on a calculator rather than on paper.

> *Vicky.* Fifty-five ... I added the two, which makes ... the two twos, which makes forty ... and then eight ... and a three and a four makes seven, so I took the two and added it on there, and then added the three and the two. [*Interviewer interjects*] 'Cos four and three makes seven ... so I put ... I took a two from the four and put it on the eight, and then I added the three and the two.

Vicky appears to have developed a more sophisticated understanding of the structure of the number word system. She is aware that the 2 in 28 stands for 'twenty', and interprets the numeral in that way rather than as 'two tens'. She makes use of this knowledge when she says 'the two twos, which makes forty' – an incorrect statement out of context, but perfectly acceptable when seen as an abbreviated articulation of a fleeting mental calculation. There were two instances of different children using this abbreviated naming of the twenties. Vicky also employs an idiosyncratic version of the 'bridging through ten' strategy: she splits the seven into a four and a three, and then into two twos and a three so that she can use one of the twos to make the 48 into 40. She then adds the remaining two to the three before combining the result with the newly formed fifty.

Similar findings to these are reported by Carraher *et al.* (1987), who undertook a qualitative analysis of the oral calculation strategies of Brazilian third-grade children, ranging in age from 8 to 13. They refer to the specific oral procedures used by the children in their sample as *heuristics*, in order to emphasise the flexibility of the children's solution strategies. They identified two types of heuristic, which they named *decomposing* (used primarily of addition and subtraction) and *repeated grouping* (used mainly for multiplication and division). The decomposing heuristic involved working with quantities which were smaller than those mentioned in the problems, and this method was used at least once by each of the sixteen children in their sample. The following clearly illustrates the heuristic:

> *Lucia* (*calculating 200 – 35*). If it were thirty, then the result would be seventy. But it is thirty-five. So it's sixty-five; one hundred and sixty-five.

Lucia decomposed the 35 into 30 and 5 – a procedure that allows her to operate initially with only hundreds and tens; the units were taken into account afterwards.

Implications

Reed and Lave (1981) differentiate between two classes of strategy that they observed Liberian tailors using for calculation purposes. The researchers placed individual strategies in one or other of these two classes, depending on whether the tailors were judged to be working with quantities or working with symbols when operating mentally on numbers. All the examples illustrated above involve children 'working with quantities' rather than blindly manipulating symbols. Vicky comes the nearest to operating in pure symbolic mode, but she also shows that she is particularly adept at switching from the symbol to the quantity it represents, and vice versa.

Fuson and Briars (1990) correctly identified the *named-value* aspect of the English spoken word system but ignore another important attribute of the system – one that young children appear to recognise and make use of in their mental calculation strategies. This additional attribute involves the important idea of *partition*. Numbers can be partitioned in a variety of ways: 27 is equivalent to 11 + 16, 24 + 3, 20 + 7 and to many other partitions. What appears to be important in this context is the fact that the actual spoken system of two-digit number words in English is itself *partitionable*. As a number is said, the words used to identify that particular number separate it into two discrete parts: a multiple of ten and a single-digit number. For example, the number 27, when expressed in words, is partitioned into a 'twenty' and a 'seven', and not into 'two tens' and 'seven', as it would be in Asian languages. In other words, a two-digit number is partitioned into *quantities* in English rather than into its *named value* components, as it would be in, say, Taiwanese.

As discussed earlier, Fuson and Briars (1990) recommend that young children progress from working with numbers up to eighteen to operating with four-digit numbers. This is to 'protect' them from dealing with the two-digit numbers which are not *named-value*. This jump from operating with small numbers to calculating with those in the thousands would appear to challenge the accepted approach to progression. I would like to argue that it might be better to take advantage of the fact that the language associated with two-digit numbers between twenty and ninety-nine is *partitionable*, and to concentrate on developing children's ability to use this aspect in order to extend and refine their mental calculation skills. Using Reed and Lave's (1981) terminology, capitalising on this *partitionable* aspect of the English counting word system would allow the children to add and subtract quantities rather than add and subtract digits.

Fuson and Briars (1990) also recommend that children should work with Dienes blocks to model calculation with four-digit numbers, as that will enable them to construct the appropriate integrated conceptual structures. Their argument is that, having worked with the equipment and the written algorithm in a context where the overt relationships can be discerned, children will be able to generalise when working with two-digit numbers, and will not become confused by the unorthodox system of naming this category of numbers.

There is no doubt that, from the point of view of someone who understands the standard algorithm for addition, Dienes base-ten blocks appear to constitute an appropriate model for our place value system and related written algorithms. However, there is an in-built assumption in Fuson and Briars (1990) that children who work with this apparatus will make the necessary connections between the practical activity they are carrying out with the materials and the procedures of the algorithm they are setting out on paper. Evidence of this transfer is actually very tenuous. As part of a larger project looking at the teaching of mathematics to children in the middle years, Hart (1989) observed lessons where teachers were about to take what she called the 'formalisation' step. This was where the mathematical ideas being taught were to be developed from the practical activities undertaken in earlier lessons to a generalisable procedure. In the specific case of subtraction it meant progressing from the practical use of base-ten blocks to the development of the decomposition written algorithm for subtraction. Hart (1989) concluded that children did not actually appear to make the connection between the practical and the symbolic representations of the subtraction algorithm.

Given the results of Hart (1989), it may be useful to focus on the 'partitionable' aspects of our system of counting words and search for an alternative model. The illustrative examples of children's work discussed above suggest that the mental methods children use allow them to work them from left to right, following the natural procedure for reading numbers, and help them keep track of their mental processes, whilst at the same time enabling them to obtain a useful first approximation to the final answer. Dienes base-ten blocks, on the other hand, model the standard algorithms, where children are expected to work from right to left, combining the units first and then exchanging for a ten in the case of addition. The blocks therefore oblige children to perform mental addition based on the standard written 'carrying' or 'exchanging' algorithms.

I would argue that we need to find a model for teaching place value which relates more closely to the language used to express two-digit numbers in the English spoken system of number words. Alternative approaches to the development of appropriate conceptual structures have been suggested (Beishuizen, 1993; Wigley, 1997). It is time British researchers attempted an evaluation of these alternative models.

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