Educational Review, Vol. 52, No. 3, 2000

Teaching Place Value in the UK: time for a reappraisal?

IAN THOMPSON, *Department of Education, Newcastle upon Tyne, UK*

ABSTRACT *This article takes the form of a critical appraisal of what is traditionally considered to be one of the most important concepts in the teaching and learning of number in primary school: place value. A consideration of the concept from a variety of perspectives suggests that the concept is too sophisticated for many young children to grasp. Recent research findings on the range of mental calculation strategies used by children for two-digit addition and subtraction and some of the recommendations of the National Numeracy Strategy* Framework for Teaching Mathematics from Reception to Year 6 *are discussed. The conclusion drawn is that there is a need for a reappraisal of the teaching of place value in primary school.*

In 1985 Hilary Shuard wrote (p. 114):

> in the present curriculum the introduction of ideas of place value is probably postponed for too long for many children.

In this article I shall argue that research conducted since this was written suggests that the opposite is more likely to be the case: most children are taught place value at too early an age and continue to be confused by it for a long time.

We tend to forget that the concept of place value was a late arrival in the development of number notation. The seemingly simple notion of using a limited number of symbols combined with the innovatory idea of allocating meaning to the position of these symbols in a numeral caused a revolution in the development of numerical calculation. Menninger (1969) expresses his amazement at the fact that medieval Europe never developed a named place-value notation, despite the fact that this was not only at hand in visual form on the counting board but was heard daily in the spoken number sequence. The fact that it took such a long time for humankind to invent this important idea should signal the fact that people are going to find the concept quite difficult.

Research on Place Value

A substantial amount of research was carried out in the late 1970s and early 1980s on children's understanding of the place value concept. The fact that 86% of 11-year-olds succeeded in selecting the number worth seven tens from a list of different numbers (Assessment of Performance Unit (APU), 1980) might suggest that there is no real problem since the majority of children appear to understand the

ISSN 0013–1911 (print)/ISSN 1465–3397 (online)/00/030291-08 © 2000 Educational Review

concept. However, further exploration reveals a somewhat different story. Two important aspects of place value that we might expect someone who understands the concept to have grasped are the relationship between adjacent columns of a multi-digit number and the specific way in which a number ending in 99 changes when one more is added. Interview research conducted by the Assessment of Performance Unit (APU, 1982), involving 222 11-year-olds, used a notation board with three columns labelled 100, 10 and 1. After a simple familiarisation activity acquainting the children with the relevance of the board to the place value system two cubes were placed in the units column and were then moved to the tens column. The children were asked to say how the value of the cubes had changed. Only 36% indicated that it had increased ten times.

Ward (1979) asked 10-year old children to state the next number that would appear on a meter currently showing 06299, and found that only 41% gave the correct answer. A similar question, described by Brown (1981), was used as the basis for the famous Cockcroft '7-year gap' slogan: the oft-repeated observation that there could be as much as a 7-year difference in the range of understanding at age 11 of certain concepts. The Concepts in Secondary Mathematics and Science research project (Hart, 1981) tested 10,000 children between the ages of 12 and 15. On one particular place value question the children were told that the 2 in 5214 stood for 2 *hundreds*, and were then asked what the 2 stood for in 521,400. Only 22% of the 12-year-olds and 31% of the 14-year-olds gave the correct answer (Brown, 1981).

More recent research has involved a study of the effects of language factors on place value understanding (Miura *et al.*, 1993). Children who speak English were compared with children who speak Asian and other languages. The Asian-language speakers were found to have a better understanding of place value than speakers of other languages. Young-Loveridge (1999) suggests a range of reasons as to why this might be the case. Fuson *et al.* (1997) have developed an elaborate multi-stage model for the development of an understanding of place value.

The research discussed above would appear to add support to the argument that place value is a highly sophisticated concept that is not really understood by many children even at the end of their primary schooling.

A Theoretical Perspective

In the early 1960s Zoltan Dienes developed his Multibase Arithmetic Blocks (MAB) and propounded a six-part theory of mathematics learning which provided an underpinning rationale for their use (Dienes, 1960). These blocks were seen at that time by many mathematics educators as the ideal equipment for helping children construct an understanding of place value in a practical context. HMI (Department of Education and Science (DES), 1979) explained that using such blocks in different bases:

> ... facilitates the abstraction of those mathematical and notational generalisations underlying different number systems, and this, in turn, enables the child to understand the operation of these generalisations in the decimal system. (p. 82)

This statement by HMI describes the *mathematical variability principle* of Dienes' theory. But however appealing this theory might have appeared at the time, the dearth of research evidence that does exist (Biggs, 1967) is fairly inconclusive. On the other hand, many of the teachers who used the apparatus found that working in different bases tended to confuse rather than clarify the issue, and as a result many of them stopped using MAB. By 1989 HMI also appeared to be less enthusiastic about the multi-base aspect of the equipment, stating simply that:

> The carefully planned use of commercially produced structured apparatus to model the number system enabled many children to see more easily how the system works. (DES, 1989, p. 16)

This statement was situated in the report next to a photograph of young children working with base-10 equipment. All reference to multi-base work appears to have been dropped.

Before looking at the role of place value in the learning of mental and written calculation methods, one particular aspect of the approach taken to the teaching of number in Germany, Switzerland and the Netherlands will be compared with the approach taken in England.

Approaches to Teaching Number

The teaching of primary school number in many European countries is seen by researchers and teachers alike as comprising two distinct phases: a period up to about 7 years of age which is spent gaining a thorough grasp of addition and subtraction facts to 20 and a more protracted second phase working with numbers from 20 to 100 (Bierhoff, 1996; Beishuizen, 1999). This work with numbers to 20 is intensive and comprehensive, and children are expected to learn their addition and subtraction bonds thoroughly. The second phase continues by building on the facts learned in the first stage in a carefully structured manner. Familiarisation with the new range from 20 to 100 involves preparatory work with multiples of 10 which are to serve as 'landmarks' within this range.

Number work in England is also traditionally seen as comprising two phases: an initial introduction to mental and written calculation with single digits followed by a substantial period of work on 'tens and units'. In fact, a very influential book from the 1970s advised teachers that: 'as soon as numbers greater than ten need to be written the first introduction to the structure of our notation has to be made' (Williams & Shuard, 1976, p. 120).

A look at any commercial primary mathematics scheme, old or new, will show that this has traditionally been, and to a certain extent, still is, the standard recommended approach to the teaching of two-digit number work. This work usually involves children in a range of activities, such as:

- grouping games;
- discussion of numbers in terms of 'tens and units';
- constructing numbers using base-10 equipment;
- representing numbers on an abacus;
- working with columns headed T and U in exercise books;
- doing paper and pencil additions and subtractions, etc.

This means that in England we introduce children to place value concepts as soon as they encounter two-digit numbers, even though these numbers happen to be the 'awkward' teens. It is unfortunate that, just as children are struggling to make sense of the confusing set of names we give to these numbers, they are also introduced to one of the most sophisticated concepts in early mathematics. It must be difficult enough for a 6-year-old who is trying to discern some pattern in the number names to have to wrestle with the fact that even though 23 (twenty-three) is 'twenty and three', 15 is read as 'fifteen'. In this case not only is the number read from right to left, with the five named first, but the five becomes 'fif' and the ten becomes 'teen'. Putting an additional layer of sophistication on top of this, namely, that the 1 goes in the 'Tens' column and the 5 goes in the 'Units' column, can only confuse the child even more. This difference in approach means that in the English system 15 is treated in a similar way to 51, whereas in other European countries it is treated very differently.

Reconceptualising Place Value

There would appear to be sufficient evidence to suggest that children do not need to learn about place value until they are expected to perform standard written algorithms for the basic operations: currently a goal for all children by the end of their primary schooling in England. It needs to be made clear that what is meant by place value in this article is the standard, traditional interpretation as emphasised in all schemes of work, and which involves terminology such as 'grouping in tens', 'the hundreds column', 'four tens and six units', etc. 'Place value' is taken to mean the value assigned to a digit according to its position in a number—described in the Cockcroft Report (DES, 1982) as:

> ... the very important concept of place value (that is, for example, that the 2 stands for 2 units in the number 52, for 2 tens in the number 127 and for 2 hundreds in the number 263). (p. 87)

A close study of the substantial body of research into children's idiosyncratic mental strategies suggests that it is possible to isolate a concept that precedes place value in the learning sequence, and which is less formal and more readily understood by young children. As yet, this concept does not have a name, although a suitable descriptor might be 'quantity value' (based on Reed and Lave's (1981) distinction between calculation strategies which involve the manipulation of quantities and those which involve the manipulation of symbols). 'Quantity value' would refer to the actual quantities represented by each of the digits in a given written number. For example, in 365 the 6 stands for 60 and the 3 for 300. The making of this seemingly fine distinction might appear to be little more than hair-splitting, but a consideration of the nature of mental calculation strategies and written computation algorithms should show that this distinction is, in fact, a crucial one.

'Quantity Value' and Mental Calculation

It is generally accepted in the research literature that the two most common mental strategies used by children for two-digit addition and subtraction are the 'split' method (partitioning) and the 'jump' method (sequencing) (Beishuizen, 1993; Thompson & Smith, 1999). A child using the 'split' procedure would work out $47 + 36$ in the following way: $40 + 30 = 70$; $7 + 6 = 13$; $70 + 13 = 83$. A subtraction such as $83 - 47$, solved using the 'jump' method would be calculated as $83 - 40 = 43$; $43 - 7 = 36$. A key procedure common to both strategies is 'partitioning': the splitting of two-digit numbers into the quantities represented by the number names. So, 47 (forty-seven) is partitioned into 40 (forty) and 7 (seven), and not into '4 in the tens column and 7 in the units column' or even '4 tens and 7 units'. In the 'split' method *both* numbers are partitioned, whereas in the 'jump' method only one of them is, and chunks of this partitioned number are added to or subtracted from the other in a sequential manner.

Two other less common mental strategies used by children are 'complementary addition', usually used for difference problems, and 'compensation' for adding or subtracting numbers ending in 7, 8 or 9. Using the former method the difference between 45 and 27 would be calculated as '27 to 30 (3); 30 to 40 (10); 40 to 45 (5), and the three steps 3, 10 and 5 would be added to give 18. At this point some form of 'jotting', either in numerical form or using an empty number line, might be made to help in remembering the size of the steps taken. Using compensation, $45 - 27$ would be calculated as $45 - 30 = 15$ and $15 + 3 = 18$. In this case 30 has been subtracted instead of 27, and so the extra 3 that has been taken away has to be added back at the end of the calculation. It is important to observe that none of these four strategies involves the use of the formal place value concept.

Two of the strategies recommended in the National Numeracy Strategy *Framework for Teaching Mathematics* (Department for Education and Employment (DfEE), 1999) for mental multiplication and division are doubling/halving and partitioning. The latter is a generalised version of the former, making use of the distributive property: $a \times (b + c) = a \times b + a \times c$. So, for example, to double 36, children would partition 36 into 30 and 6, double the 30, double the 6 and then add the two answers together. Halving 48 would be done by halving the 40 and then halving the 8. Finding a quarter of 48 would involve halving twice. In Year 6, children are expected to be able to calculate 86×7 mentally by adding 80×7 to 6×7. These mental multiplication and division strategies involve 'partitioning'—a procedure heavily dependent upon an understanding of 'quantity value' and not 'place value'.

'Quantity Value' and Written Calculation

A consideration of the 'informal' written strategies described in the *Framework* for the basic operations at Key Stage 2 confirms the importance of 'quantity value' even at this level. The recommended algorithm for addition is described as 'Adding the most significant digits first', and an example given for Year 4 is:

$$\begin{array}{r} 367 \\ +\ \underline{85} \\ 300 \\ 140 \\ \underline{12} \\ \underline{452} \end{array}$$

This algorithm constitutes an almost exact model of the partitioning mental strategy described above: the digits are treated as quantities; they are added from the left according to their significance; they provide a good first approximation to the answer; and there is no 'carrying' (a procedure that necessitates an understanding of column-based place value). In a similar way, no 'exchanging' is involved in the recommended informal subtraction methods.

For multiplication the informal 'grid method' and a slightly more formal written version are recommended for Year 4 children:

$$\begin{array}{rl} 23 & \\ \times\ \underline{8} & \\ 160 & 20\times 8 \\ \underline{24} & 3\times 8 \\ \underline{184} & \end{array}$$

Even though the algorithm is set out in column format the actual calculation does not involve place value: the 2 in 23 is treated as 'twenty' not 'two'. In fact there is no reason at all why the algorithm needs to be set out vertically—other than to prepare children for the standard algorithm for multiplying pairs of two-digit numbers in Year 5.

For written division both the recommended 'informal' and the 'standard' methods involve subtracting chunks of the divisor from the number being divided. The following example is taken from the 'Standard written methods' section for Year 5:

$$\begin{array}{r} 6)196 \\ -\ \underline{180} \\ 16 \\ \underline{12} \\ \underline{4} \end{array}$$

Answer 32 remainder 4

This 'chunking' procedure, which can be used at different levels of sophistication, demands good estimation skills and the ability to make effective use of addition, subtraction and multiplication knowledge. An understanding of quantity value would seem to be more useful than place value for the efficient execution of this method.

Conclusion

The available evidence suggests that place value is a concept which children not only have great difficulty in understanding but which they also do not even need to use for mental or informal written calculations until Year 4. And yet, despite this, we insist on teaching the concept from Year 1. It could be argued that attempts to teach place value as early as we currently do are counter-productive, and that this premature emphasis has been a contributing factor in our relatively poor performance in the number section of international surveys.

Too early an introduction to formal place value can lead to children developing an impoverished mental representation of two-digit numbers, where they always visualise them set out in columns. Mental calculation with such numbers then becomes little more than the execution of the standard written algorithms (often incorrectly) in the head: what Fuson (1992) calls 'using a concatenated single-digit conceptual structure for multi-digit numbers' (p. 262).

Kamii (1985) reports that first graders (Year 2) often have no difficulty in counting out the correct number of objects when shown the numeral 16 nor in writing this numeral when shown sixteen objects. Also, when asked 'What does this part [the 6 in 16] show?' they usually produce the correct number of objects. However, when asked a similar question with respect to the digit 1 in the numeral 16 the results deteriorate substantially. Almost no first graders responded correctly to this question in her research, and two-thirds of Grade 3 (Year 4) and one-half of Grade 4 children incorrectly answered the question by referring to a single object rather than ten.

This research suggests that children are still able to work successfully with two-digit numbers, including the teens, without being explicitly aware that the first digit stands for the number of tens. This situation is analogous to the findings of Pennington *et al.* (1980) that over 70% of the 5- and 6-year-olds who had been unsuccessful on a Piagetian number conservation test were able to make accurate judgements of equivalence when they used counting. However, further research is needed to ascertain whether children whose understanding of formal place value is poor are still able to calculate sums and differences involving two-digit numbers in their heads.

There is almost universal agreement about the need to concentrate on teaching mental calculation methods before written algorithms. In order for children to develop the skills and understandings necessary for successful mental calculation they need to have a good understanding of 'quantity value'. Consequently, this concept needs to be taught much earlier than the more formal place value concept. We have recently witnessed a reassessment of the way in which we teach early number in this country (Thompson, 1997). This article is an attempt to provoke a reappraisal of the teaching of place value.

Correspondence: Ian Thompson, Department of Education, St Thomas Street, Newcastle upon Tyne NE1 7RU, UK.

REFERENCES

APU (1980) *Mathematical Development: Primary Survey Report No. 1* (London, HMSO).

APU (1982) *Mathematical Development: Primary Survey Report No. 3* (London, HMSO).

BEISHUIZEN, M. (1993) Mental strategies and materials or models for addition and subtraction up to 100 in Dutch second-graders, *Journal for Research in Mathematics Education*, 24(4), pp. 294–323.

BEISHUIZEN, M. (1999) The empty number line as a new model, in: I. THOMPSON (Ed.) *Issues in Teaching Numeracy in Primary Schools* (Buckingham, Open University Press).

BIERHOFF, H. (1996) *Laying the Foundation of Numeracy: a comparison of primary school textbooks in Britain, Germany and Switzerland* (London, National Institute for Economic and Social Research).

BIGGS, J.B. (1967) *Mathematics and the Conditions of Learning* (Slough, NFER).

BROWN, M. (1981) Place value and decimals, in: K. Hart (Ed.) *Children's Understanding of Mathematics: 11–16* (London, John Murray).

DES (1979) *Mathematics 5–11: a handbook of suggestions*, HMI Matters for Discussion 9 (London, HMSO).

DES (1982) *Mathematics Counts* (Cockcroft Report) (London, HMSO).

DES (1989) *The Teaching and Learning of Mathematics* (London, HMSO).

DfEE (1999) *Framework for Teaching Mathematics from Reception to Year 6* (London, DfEE).

DIENES, Z.P. (1960) *Building up Mathematics* (London, Hutchinson).

FUSON, K.C. (1992) Research on whole number addition and subtraction, in: D.A. GROUWS (Ed.) *Handbook of Research on Mathematics Teaching and Learning* (New York, Macmillan).

FUSON, K.C., WEARNE, D., HIEBERT, J., HUMAN, P., MURRAY, H., OLIVIER, A., CARPENTER, T.P. & FENNEMA, E. (1997) Children's conceptual structures for multidigit numbers and methods of multidigit addition and subtraction, *Journal for Research in Mathematics Education*, 28(2), pp. 130–162.

HART, K.M. (1981) *Children's Understanding of Mathematics* (London, John Murray).

KAMII, K. (1985) *Young Children Reinvent Arithmetic: implications of Piaget's theory* (New York, Teachers College Press).

MENNINGER, K. (1969) *Number Words and Number Symbols* (Massachusetts, MIT Press).

MIURA, I.T., OKAMOTO, Y., KIM, C., STEERE, M. & FAYOL, M. (1993) First graders' cognitive representation of number and understanding of place value: cross-national comparisons—France, Japan, Korea, Sweden and the United states, *Journal of Educational Psychology*, 85, pp. 24–30.

PENNINGTON, B.F., WALLACH, L. & WALLACH, M.A. (1980) Non-conservers' use and understanding of number and arithmetic, *Genetic Psychology Monographs*, 101, pp. 231–243.

REED, H. J. & LAVE, J. (1981) Arithmetic as a tool for investigating relations between culture and cognition, in: R.W. CASSON (Ed.) *Language, Culture and Cognition: anthropological perspectives* (New York, Macmillan).

SHUARD, H. (1985) *Primary Mathematics Today and Tomorrow* (London, Longman).

THOMPSON, I. (1997) The early years number curriculum today, in: I. THOMPSON (Ed.) *Teaching and Learning Early Number* (Buckingham, Open University Press).

THOMPSON, I. & SMITH, F. (1999) *Mental Calculation Strategies for the Addition and Subtraction of 2-digit Numbers*, Report for the Nuffield Foundation (Newcastle upon Tyne, Department of Education, University of Newcastle upon Tyne).

WARD, M. (1979) *Mathematics and the 10-year-old* (London, Evans/Methuen Educational).

WILLIAMS, E.M. & SHUARD, H. (1976) *Primary Mathematics Today* (London, Longman).

YOUNG-LOVERIDGE, J. (1999) The development of place value understanding, in A. ELSE (Ed.) *Exploring Issues in Mathematics Education: proceedings of a research seminar on mathematics education (Year 0–6 students)* (Wellington, Ministry of Education).