Teaching mental calculation

In introducing some suggestions for improving our teaching strategies, Ian Thompson reminds us that one of the four key principles underpinning the National Numeracy Strategy (NNS) is "an emphasis on mental calculation".

Most mathematics educators would, I think, agree that young children are now generally much more confident and competent at mental calculation than they were before the introduction of the NNS. For example, it is quite common nowadays to observe situations like the following:

- a Reception child finding 'how many beads there are altogether when three more beads are added to the five on the table' by counting on, saying "5… 6, 7, 8";
- a Year 1 child finding 7 + 6 by stating that, "Six and six makes twelve, so it's one more… thirteen";
- a Y2 child calculating 23 - 18 by saying "It's two to 20 and three to 23… so, it's five".

My own research, and that of others, suggests that only a few children made regular use of such strategies before the advent of the National Numeracy Strategy.

In this article I want to take a close look at a calculation strategy that is different from those mentioned above, with the aim of suggesting improvements in the way that this strategy is currently presented in many teaching schemes, and consequently, in the way that it is taught in the vast majority of schools. The NNS *Framework* introduces the strategy in Year 1 and then develops it throughout Key Stage 1 via the objective 'Add or subtract 9, 19, 29… or 11, 21, 31…by adding or subtracting 10, 20, 30… and adjusting by 1'. In Key Stage 2 the strategy becomes 'Add or subtract the nearest multiple of 10 and adjust', only to reappear, renamed, in the context of written calculations as *compensation*.

In connection with the first part of the KS1 objective – the addition of 9, 19, 29… - I would like to mention some research I carried out that involved interviewing 144 seven-year-olds about their mental calculation methods. Interestingly, only one child actually added nine by first adding ten and then taking away one. This research, carried out before the NNS was developed, led me to conclude that the strategy involved was not a 'natural' strategy, by which I meant that it was 'unlikely to be invented by children' (though it was not as 'non-natural' a strategy as decomposition, which, to my knowledge, no child has ever invented!). My findings seemed to suggest that teaching this strategy with understanding might prove more difficult than one would think.

What I particularly want to question about the teaching of this strategy is the wisdom of working on 'adding 9 by adding 10 and *subtracting* one' at the same time as 'adding 11 by adding 10 and *adding* one', as is suggested in all the published materials that I have seen. At first glance it might seem a sensible way of proceeding given the ease with which both procedures can be demonstrated or modelled on the 100-square (see Fig. 1).

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

24+19 =
24+20-1 = 43

56+21=
56+20+1 = 77

Figure 1

However, what usually happens when these two strategies are taught together is that many children (and, in my experience, some trainees) have difficulty deciding which way to move in the second part of the calculation, and this often leads to an incorrect answer. This situation is exacerbated when the *subtraction* of 9 and 11 is introduced later via subtracting 10 and making the relevant adjustment. (Answer quickly: After moving vertically on the 100-square, do you then move to the left or the right when subtracting 9?)

So, when children have been taught the addition and subtraction of numbers ending in 9 and 1 they will have met the following four situations, illustrated here using the empty number line:

Adding 29

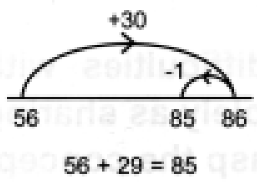

Figure 2

Adding 31

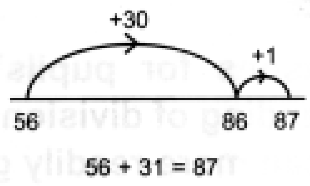

Figure 3

Subtracting 29

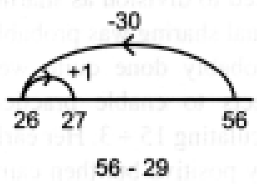

Figure 4

Subtracting 31

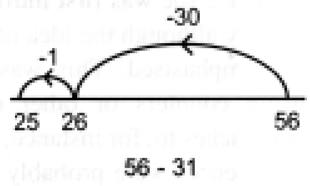

Figure 5

Is it any wonder that children have difficulty remembering which way to jump?

To my mind the main source of the problem is the *Framework*, with its insistence on linking the strategy involved in adding numbers ending in 9 with the strategy for adding numbers ending in 1, whereas in actual fact they are fundamentally different strategies. Adding 21, for example, should be linked with adding 22, 23, 24 or any number in the twenties. It is part of the basic sequential strategy for adding two-digit numbers: when calculating 27 + 24, you partition 24 into 20 and 4; add the 20 to the 27 - giving 47 - and then add the 4 to get 51. On the other hand, adding numbers ending in 9 (or 8) involves a more complex strategy: rather than adding separate parts of the number, as you would when you add, say, 31, you are actually adding a different number from the one you have been asked to add. When adding 29, you do ***not*** partition the number, you add 30 instead, ensuring that you remember to compensate by taking away one at the end of the calculation. This is conceptually a very different procedure.

I believe that this idea of adding a different number demands a great deal of confidence on the part of the children, and that this is probably the reason why only one child used this strategy in my research. My solution to the 'confusion problem' would be to teach the strategies involved in Figures 2 and 4 first of all, as part of the overall strategy for the addition and subtraction of two-digit numbers. This means that there should be no worry about which way to jump when operating on numbers ending in 1 – it's just the same as adding or subtracting any other two-digit numbers.

At a later stage I would teach the addition of numbers ending in 9 utilising the empty number line, and would follow this up almost immediately by teaching the subtraction of similar numbers. Working in this way enables teacher and pupils to focus on the *compensation* aspect of the strategy: after the initial tens jump the child has to compensate for 'over-jumping' by moving one unit in the opposite direction. This compensatory jumping process is the same whether the operation in question is addition or subtraction.

Also, treating adding and subtracting 9, 19, 29… (and even 8, 18, 28…) as a different strategy from working with numbers ending in the digits from 1 to 7 also means that children can be introduced to the correct terminology for the procedure, thus providing continuity with work in Key Stage 2 where a written version of compensation is introduced.

To summarise:

- teach the addition of 2-digit numbers ending in 1, 2, 3, 4, 5… (preferably on the empty number line) by first partitioning the number; adding the multiple of ten; and then adding the units;
- teach subtraction in the same way, subtracting the multiple of ten and then the units;
- when children are familiar with the direction of the jumps involved in addition and subtraction, introduce the addition and subtraction of numbers ending in 9 or 8 by adding the next multiple of ten, and then emphasising the compensation aspect which necessitates a jump in the opposite direction

If the *Framework* ever has to be revised or updated, I hope that the link between adding numbers ending in nine and numbers ending in one is removed, and that the differences between the two calculation strategies involved are clarified.

An earlier draft version of this article appeared in the *Times Educational Supplement*

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The challenge of poverty

Over a billion with insufficient food, no access to health care, education, clean water or sanitation. Nearly half of the world are very poor, living on less than the equivalent of $2 dollars per day for all their needs.

Clare Short, *An Honourable Deception? New Labour, Iraq and the Misuse of Power*, London: The Free Press, 2004