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PUTTING PLACE VALUE IN ITS PLACE

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A*: What happens when you multiply a number by ten?*
B: *You add a zero.*
A: *No, we say that 'the digits move one place to the left*.

The context for this exchange could be any one of several scenarios: a primary or secondary classroom, a teacher training establishment, an LEA CPD course venue or a national numeracy strategy consultants' training session. In each context the 'teacher' would be keen to avoid the acquisition of any misconceptions on the part of the 'learner' because, as we all know, the 'adding a zero' rule breaks down in the case of decimal fractions. I would like to argue in this article that the 'teacher' may well be acting in too pedantic a manner in this situation, particularly if the 'learners' are young children. I have found that the 'digits moving one place to the left or right' mantra has become more commonplace since the advent of the NNS.

Let us try to unpick what is involved in an understanding of the concept of 'moving one place to the left'. First of all, children need to realize that in mathematics we impose an invisible column structure on multi-digit numbers (unlike in literacy where a word like 'cat', even though it comprises three individual letters, does not have such a structure imposed on it). They also need to appreciate the following:

- that the quantities represented by the individual digits are determined by their positions in the number;
- that as the digits are moved to the left their value increases by an appropriate power of ten (depending on the number of places moved);
- that the value of an individual digit is found by multiplying its face value by the value assigned to its position;
- that the quantity represented by the whole number is the sum of the values of these individual digits.

These properties of place value understanding are described by Ross [1] respectively as the *positional*, *base-ten, multiplicative* and *additive* properties of place value. They obviously demand a fairly sophisticated level of understanding.

So, what do we currently know about young children's ability to meet this demand? A recent investigation by Thompson and Bramald [2] explored the relationship between young children's understanding of place value and their ability to add two-digit numbers. The study took the form of a series of one-to-one interviews with a sample of 144 children aged 6 to 9 from eight primary schools. On the oral question 'What is 25 plus 23' (accompanied by a card with 25+23 written on it), 63% of the sample answered the question correctly using a

strategy that partitioned either (or both) 25 and 23 into 20 and 5 and 20 and 3 respectively. However, on two other practical questions that involved important aspects of place value the children were less successful. On the 'milometer' question (Brown, [3]) [*The reading is 6299, what will it be after the car has traveled one more mile?*], 24% gave the correct answer. On the 'bricks' question (APU, [4]) [*How does the value of a brick change when it is moved one column to the left?*] only 10% were correct. The total number of children who were successful on both questions was just **4** (all from Y4), compared to the **91** who performed the addition correctly using partitioning. An important conclusion of this study was that children are able to add two-digit numbers successfully using partitioning without needing to have an understanding of place value.

In the context of this article it seems fair to conclude that children in Y2 and Y3 have little no understanding of what we conventionally call place value. In Thompson's [5] terms they have some appreciation of the 'quantity value' aspect of place value (73 is 70 plus 3), but not the 'column value' aspect (73 is 7 in the tens column and 3 in the units column). Consequently, it would appear to be somewhat over-optimistic to expect young children to understand the concept of 'moving the digits one place to the left', which, as argued above, appears to demand a fairly sophisticated understanding of place value[1].

There are other instances of mixed or confusing messages concerning aspects of place value in the *Framework* [5]. For example, whenever 'know the value of each digit in a number' is mentioned it is exemplified in terms of '623 is 600 + 20 + 3' (quantity value) rather than in terms of 6 hundreds, 2 tens and 3 ones (column value). However, the latter interpretation is emphasized when the *Framework* suggests that children as young as 6 or 7 should be able to 'represent 14 on an abacus': surely one of the most formal representations of the column aspect of place value. And, why on earth is the word 'exchange' listed in the Y1 section of *Mathematical vocabulary* [8] when written methods for addition introduced as late as Y4 are described as being only about '*preparing* for carrying' (my italics)?

The reader may well want to argue that if children are taught to 'add a zero' they will be learning a 'rule' that they will have to unlearn later. My response to this would be that:
1. We already do this in the area of calculation and the number line: initially a young child 'cannot take five from three' nor 'divide 3 by five', and much later a secondary pupil 'cannot find the square root of negative four'.
2. Much current emphasis on errors and misconceptions focuses on 'cognitive conflict' (see Swan [9]). The 'add a zero' issue provides a perfect context for such discussion work.
3. I would also like to see some research (rather than anecdotal) evidence that might provide information about the percentage of children in Y6 (the first time multiplying a decimal by 10 appears in the *Framework*) who are likely to write 0.8 x 10 = 0.80. Personally, I would put money on this percentage being very low!

In conclusion, the 144 children in the Thompson and Bramald [2] research were asked what 8 x 10 was, and were then asked '*What happens when you multiply a number by 10?* The first part was answered correctly by 85% of the children. The second part generated a range of answers: 18% of the sample were unable to answer the question; 50% of Y3 and 63% of Y4 said *'Add a nought (or zero)'*; others said: *A tens number; It has to be a number in the tens; It highers it; Count up in tens; You come up with the –ty of that number; It changes the front number;* and *You share it (!).* Not a single child mentioned 'moving places', 'shifting into columns' or 'place holders' despite the fact that many of the teachers interviewed said they had addressed the topic using such language.

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Footnote

[1]Exploring the 'moving digits' slogan further we find that the situation is more complex than it at first seemed. It is not enough just to say that the digits move, given that, visually, they appear to be no different after this movement. What we obviously have to do is signify that they have indeed been moved. So, perhaps we should change A's response above to: *No, we say the digits move one place to the left, and then we insert zero as a place holder.* This action would seem to be supported by the NNS *Framework* [6], which includes the following Key Objective in its yearly teaching programme for Y2 children: *Know what each digit in a two-digit number represents, including zero as a place holder* (p. 10), and the National Curriculum [7] which states that Key Stage 1 children should be able to: *recognize that the position of a digit gives its value and know what each digit represents, including zero as a place holder* (p. 63). So, an appreciation of the concept of 'place holder' is prescribed for children aged six or seven. However, having explained the shifting of digits and the use of a place holder, it would be difficult for a teacher not to smile at the 'smart Alec' response from 'B'; *So, as I said, to multiply by 10 you add a zero*!

References

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