

Fig. 1. Partitioning sets

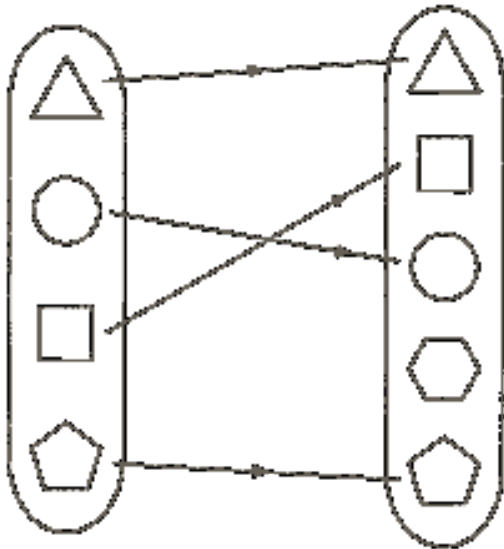


Fig. 2. Matching sets

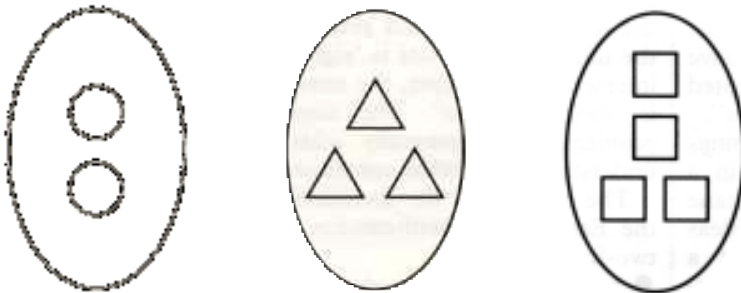


Fig. 3. Ordering sets

# “Pre-Number Activities” and the Early Years Number Curriculum

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## The Current Situation

Most of the currently available commercial mathematics schemes aimed at Key Stage 1 children appear to follow a stereotyped sequence, particularly with respect to their approach to early number. Almost without exception the schemes contain a substantial collection of activities for children to engage with before they begin to tackle work involving actual numbers. These activities have come to be known as “pre-number activities”, and are considered to be essential pre-requisites to basic number work.

Whilst engaging in these activities children classify objects and sort sets according to different criteria. They partition these sets in a variety of ways (Fig. 1) to create mutually exclusive (or disjoint) subsets, and they work with these subsets and their complements. They perform matching activities connecting pictures of identical or related objects, and attempt to find equivalent or non-equivalent sets (Fig. 2). They carry out ordering activities continuing bead patterns, placing objects in order of size or multilink towers in order according to the number of cubes contained in each tower (Fig. 3).

Why is it that most Early Years mathematics schemes appear to follow this sequence of activities fairly rigidly? In order to investigate this question it is illuminating to consider what was happening to the subject of mathematics itself around the beginning of this century. By this time many traditional areas of mathematics had been reconceptualised. At a later date the influential Bourbaki group attempted to integrate the myriad branches of the subject by laying emphasis on the major underlying structures. Set theory with its very precise language was to become a major factor in this great unifying process.

These developments in the area of mathematics came to have a substantial effect on the content of University mathematics courses, and at a later date these changes filtered down to secondary schools with the result that the

early sixties saw the emergence in Britain of a variety of “modern mathematics” projects such as the School Mathematics Project, the Midlands Mathematics Experiment, the Scottish Mathematics Group, and various other smaller-scale projects. Most of these early secondary projects took a very rigorous approach to the teaching of the subject.

The major changes taking place in the mathematical diet of the older children led inevitably to calls for a review of the primary mathematics curriculum, and 1964 saw the launch of the “Nuffield Mathematics 5 to 13” project under the direction of Geoffrey Matthews — a very able mathematician with experience of curriculum development at the secondary level, but with little or no experience of working with primary age children. Matthews’ interest in mathematical precision and rigour is apparent in the approach taken to number by the Project, and is clearly illustrated in one of the earliest of the project’s publications — *Mathematics Begins* (1967). This succinct and lucid booklet contains a rigorous, but eminently readable, account of the basic ideas underlying the concept of number.

The decision of the Project team not to produce pupil materials (in retrospect probably a wrong decision) led to some members of the writing team leaving to write their own schemes. Harold Fletcher was one of these, and he became co-author of the famous — some might say infamous — “Mathematics for Schools” (“Fletcher Maths”) scheme which was published in 1970. A later decision to produce more materials than just guide books for teachers led to the appearance of the Nuffield Mathematics Scheme in 1979. Almost every commercial scheme that has appeared since then shows the extent to which it has been influenced, directly or indirectly, by those early Nuffield booklets or by the Project’s offshoots. Not a single scheme over the years appears to have questioned the importance, or otherwise, of the accepted approach to number via specific “pre-number activities”.

Even though it may well be the case that sets, mappings and relations are indeed important ideas involved in a rigorous analysis of the concept of number, it is germane to ask whether an understanding of these particular ideas is a necessary prerequisite for the development of a satisfactory concept of number. Ideas which are mathematically sound may not necessarily always be pedagogically sound. Take, for example, the suggestion in one of the early Nuffield publications that, because the operation of subtraction is not closed on the set of natural numbers, teachers ought to delay the teaching of subtraction until after the introduction of the integers! A later booklet makes a similar suggestion that work with fractions should be postponed until children are familiar with the set of rational numbers. Such ideas may well make mathematical sense, but it would be extremely difficult to convince teachers that they make educational sense!

## Counting and “Pre-number Activities”

There is a substantial body of research evidence which details the important role played by counting strategies in the development of children’s number understanding. This evidence suggests that counting should comprise an important part of the Early Years mathematics curriculum. Fuson (1982) has shown that children learn the difference between counting words and non-counting words at a very early age. A large group of children between the ages of two and five were asked to count collections of objects, and only two of them — both aged two — used non-counting words in their response to the task.

Gelman and Gallistel (1978) have postulated five

counting principles which they argue guide children’s early counting activities:

- the ‘one-one’ principle  
whereby every item is labelled with one and only one unique label
- the ‘stable-order’ principle  
whereby the labels used must be ordered in the same sequence each time
- the ‘cardinality’ principle  
whereby the label assigned to the last object to be counted indicates the number of objects in the collection
- the ‘abstraction’ principle  
whereby objects of any kind — large, small, real, imaginary, visible, hidden, etc. — can be counted
- the ‘order-irrelevance’ principle  
whereby objects can be counted in any order.

In the light of these counting principles there are questions which need to be asked about the sorting, matching and ordering activities described earlier and the importance of their role as pre-number activities. For example, one important attribute of a set is that its elements must have some definable property in common. However, Gelman’s fourth principle states that one can count anything at all: the objects to be counted need have nothing in common. So, in what way does the concept of a set help in the acquisition of this ‘abstraction’ principle?

If the language of sets is supposed to clarify the ideas involved, how does the definition of addition given in some teachers’ texts, namely, *Addition can be defined as the union of disjoint subsets*, achieve this clarification? Although this definition may well be mathematically sound it is of little or no help to someone who wishes to find the sum of the elements in two sets which are not disjoint! For example, a child may be trying to find the total number of letters in the two words *boys* and *girls*. The number of elements in the union of these sets is ‘eight’ since the letter *s* is in the intersection. However, the answer to the original problem is obviously ‘nine’. The *disjoint sets* definition causes problems, and generally adds nothing to a learner’s understanding of what constitutes ‘addition’.

The reasons for the inclusion of ‘matching’ activities in the Early Years’ mathematics programme appears to be two-fold:

- to help children to decide whether one set contains more or fewer elements than another, and
- to prepare them for counting (particularly Gelman’s “one-one” principle) where number names are matched in one-to-one correspondence to the objects to be counted.

Brainerd (1979) suggests that one-to-one matching between the objects of one collection and the objects of another in order to compare their size is a relatively late accomplishment. Young children appear to be more likely to count in order to find which set has more members.

A very important aspect of the counting process involves the accurate matching of number names to objects, but why should children make any connection between the physical, written activity of drawing arrows from objects in one set to objects in another and the important “one-one” principle involved in the mental activity of oral counting? It would seem much more likely that, just as they learn to talk by engaging in the process of talking — making errors, correcting these themselves or having them corrected for them — children will come to appreciate Gelman’s “one-one” principle of counting, and more, by engaging in the process of counting.

Children participate in ordering activities because the counting numbers constitute an ordered set and accurate counting involves the enunciation of the number names in the correct sequence (the “stable order” principle). This would appear to constitute a reasonable argument for the inclusion of pre-number work on ordering. However, we

find, yet again, that there is an underlying assumption that children will make a connection between continuing the order of a pattern or ordering towers (either practically or with paper and pencil) and the oral recitation of the counting words in the correct order. It is difficult to envisage that they actually will.

Learning the sequence of counting words, at least up to twenty, is essentially a serial recall task. Fuson (1982) argues that the sequence initially functions as a single, connected whole, with children unable to produce independent words from within the sequence. Later in their development they are able to do this, and the production of one word often acts as a stimulus for the recall of the next word. Apart from the production of the counting words in the correct sequence, order becomes unimportant in the activity of counting, for, as Gelman's fifth principle informs us, the actual order in which you count a set of objects is immaterial. The important idea is that no matter which order you count them in, provided that you follow Gelman's first three principles, you will always get the same correct answer.

Over the last twenty years researchers have been reporting on the number understanding that many young children possess at the beginning of the formal educational process. Aubrey (1993) details the level of understanding of a group of children (average age 4 years and 10 months) on ten separate aspects of number knowledge. The sixteen children studied were in the reception class of an urban infant school with a catchment area that reflected a wide social and ethnic mix. She found that twelve of the children were successful in counting two separate arrays of objects which contained three and seven items. Two of the other children were able to count the group of three items only. She also reported that three quarters of the children were successful in completing an activity involving an appreciation of the fact that counting an array in a variety of different orders should yield the same result. These findings suggest that many of these children, who had only been in school for just two weeks, already possessed an unexpectedly good intuitive understanding of some of the number concepts that the school was planning to teach them over the next few years!

The level of performance manifested by many of these young children raises questions concerning the content of the early years mathematics curriculum. If children are revealing these competences at the start of their school careers why do they need to be spending (wasting?) their time on such pre-number activities as sorting, classifying, matching and ordering: activities designed to prepare them for an introduction to something they already appear to understand? Underlying this traditional approach to early number is the fundamental belief that there will be a transfer of learning from work on pre-number activities to work with numbers. There appears, however to be little or no evidence to suggest that such transfer actually takes place or that this particular method of introducing work on number is any better than other possible alternatives.

## Alternative Approaches

One particular piece of research which does attempt to compare different approaches to the teaching of early number is that of Clements (1983), who devised a teaching experiment involving three groups of four-year old children. One class was taught classifying, matching and ordering skills, the second was taught rational counting skills, and the third was used as a control group. A "Number Concepts" test and a "Logical Operations" test were given as pre- and post-tests to all three groups with interesting results. As might be expected, both experimental groups outperformed the control group. There was no significant

difference in performance between the two experimental groups on the "Logical Operations" test, even though one might have expected the "pre-number activities" group to have produced a superior performance. On the "Number Concepts" test, however, the "number skills" group significantly outperformed the "logical operations" group. Clements concluded that logical operations do not necessarily constitute a prerequisite for the learning of early number concepts. I would argue that his research also suggests that a suitable alternative approach to early number is to be found in the "rational counting skills" model.

This approach would fit in well with the plethora of research evidence concerning young children's use of counting in the early stages of their learning about number. Thompson (1992) has also shown the importance of counting as a problem solving strategy with seven and eight year old children who appear not to abandon counting even when they have learned some addition, subtraction or multiplication bonds. He gives various examples of how children combine counting with other calculating skills such as the ability to work out doubles, add ten to, or subtract ten from a number, work with complements in ten both in addition and subtraction, and even use counting with known multiplication bonds in order to calculate unknown facts.

## Conclusions

There is no doubt about the extensive use that pre-school and Early Years children make of counting skills when solving problems involving simple mental calculations, particularly in situations that make "human sense" to them. This usage helps them gradually become aware of the various principles that underlie rational counting. As they progress through school they continue to use counting as an important part of their problem solving armoury, and develop creative adaptations of this basic skill by learning to combine counting with other newly-acquired mathematical skills, facts and knowledge. Number concepts and meaningful counting develop in tandem, and it is through the application of increasingly more efficient counting procedures that young children gradually discover or construct for themselves many of the basic number concepts.

The ability to use counting and its various sub-skills competently and successfully would appear to be far more crucial to a child's development of an understanding of number than the ability to sort, partition, match and order sets. Is it not time, therefore, to question the importance given to the role of "pre-number activities" in the Early Years mathematics curriculum? ☒

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