PLACE VALUE?

Ian Thompson argues it is time to redefine place value.

The Independent Review of the Primary Curriculum: Interim report (DCSF, 2008) – aka the Rose report – sets out a design for planning the primary curriculum organised through six areas of learning. However, although Recommendation 7 begins with:

> *Primary schools must continue to give priority to literacy and numeracy* … (p. 44)

there is little in the document about mathematics education.

In an attempt to address the task of reducing the total content of the primary curriculum – what it calls the 'quarts-into-pint-pots' problem – the document argues that one way to achieve this might be to focus planning for progression rigorously on a manageable set of 'key ideas' (a phrase that appears 22 times in the document!). Subjects and areas of learning are deemed to be shaped by key ideas that are essential to a child's understanding. These ideas should build on the EYFS areas of learning and development; be well matched to primary children's intellectual development; and be easily understood by generalist primary teachers.

In order to illustrate this argument specifically for mathematics, the document refers to a discussion paper prepared for the Rose Review by the Royal Society's Advisory Committee on Mathematics Education (ACME, 2008). This paper uses language popularised by the ATM in talking about 'big ideas' rather than 'key ideas'. These ideas comprise:

- Place value and the number system;
- Conservation of number and measures;
- Equivalence Relations;
- Dimensionality.

It is the first of these – place value and the number system – that I wish to discuss. ACME's (2008: 4) explanation of place value is:

> *'In our written representation a number like 305 means that the 3 is equivalent to 3 hundreds (and also to 30 tens), the zero indicates no remaining sets of ten but the 5 shows 5 remaining ones.'*

You are probably wondering how on earth anyone could take issue with this description. Consequently, one purpose of this article is to clarify why I consider this 'explanation' to be perpetuating a traditional, old-fashioned and out-of-date description of place value.

Mental calculation

Let's start by considering mental calculation strategies. Do the following addition in your head, and then reflect on the strategy you used: 37 + 24.

You probably used one of the following strategies:

"30 and 20 is 50. 7 and 4 is 11, and 50 and 11 is 61";

"37 and 20 is 57, 57 and 4 is 61";

"40 and 24 is 64, 64 take away 3 is 61".

Unless you carried out the standard written algorithm mentally, you should notice that you treated the 3 in 37 and the 2 in 24 as 'thirty' and 'twenty' respectively; not as '3 tens' and '2 tens'.

Do the same for 74 − 27. You probably used one of the following strategies:

"74 take 20 is 54, 54 take 4 is 50, 50 take 3 is 47";

"74 take 30 is 44, 44 add 3 is 47";

"27 and 3 is 30, 30 and 44 is 74. So it's 3 plus 44... 47".

As before, the 7 in 74 and the 2 in 27 were treated as 'seventy' and 'twenty'.

Now let us try mental multiplication and division. Calculate 23 × 7. The most common mental strategy used in my own research, carried out over the last 20 years, is:

"7 times 20 is 140, 7 times 3 is 21. So the answer is 161".

To carry out a division calculation like 345 ÷ 3 mentally I would anticipate your saying:

"300 divided by 3 is 100, and 45 divided by 3 is 15, so the answer is 115".

My conclusion from this simple exercise is that most (I think I would go as far as to say 'all') mental calculation strategies for the four basic operations treat the digits in the tens or hundreds column as a quantity in their own right, i.e. the 4 in 345 is 'forty' rather than 'a four in the tens column' (or even 'four tens'). I call the former the 'quantity value' aspect of place value and the latter the 'column value' aspect. 'Column value' is the only aspect alluded to in the ACME description of place value quoted above. Ruthven (1998) also found no evidence of what is normally called place value in the mental calculation methods used by young children in his research.

Written calculation

Let us now take a brief look at *informal* written calculation procedures – some of which have been recommended since at least the mid-70s (Plunkett, 1979) – and others that were a major feature of the original National Numeracy Strategy (NNS) framework (DfEE, 1999).

As was the case with mental calculation strategies, the aspect of place value that is involved in all five of the informal written calculation procedures in the box below is 'quantity value' rather than 'column value'. Neither the concept nor the language of 'nine tens' or 'five hundreds' is used in these calculations; on each occasion it is the quantity represented by the digit that is manipulated in the calculation.

Research findings

Thompson and Bramald (2002) interviewed 144 children (48 in each of Years 2 to 4 from eight primary schools) in an attempt to explore their understanding of place value. One question asked the children to calculate 25 + 23 mentally and to explain how they did it. Over three quarters were correct and 91 of the 144 (63%) children used a partitioning strategy, splitting 23 into 20 and 3 in order to perform the calculation.

Another interview item involved discussing a picture of a car dashboard showing a mileage of 6299 and asking the children what the milometer would show after the car had travelled one more mile. Only 24% were correct on this item. A further question asked them to say how the value of several cubes had changed after they had been physically moved by the researcher from the 'ones' to the 'tens' column on a base-ten board. Only 10% were correct on this item.

Since the milometer and the cubes questions both address important aspects of place value, these findings suggests that, if you take the view that a child who can be said to understand place value would be expected to be able to give correct answers to both questions, then only 4% (four per cent!) of the sample do actually understand the concept. Comparing this 4% with the 63% who used partitioning to correctly add the two 2-digit numbers suggests that two different aspects of place value are being assessed: 'quantity value' and 'column value'.

Conclusion

The argument being made in this article is simply that mental calculation strategies and informal written procedures are based on the quantity aspect of place value, whereas standard (or compact) algorithms are based on the column aspect. So, given that the Primary National Strategy (DfES, 2006) does not include compact written algorithms earlier than Year 4, teachers should not be in a rush to teach this aspect of place value, as the research described above suggests that young children find the concept difficult. It is only when the teacher has made the decision to move children on from an informal to a formal compact written algorithm that an understanding of 'column value' is required. It is in preparing for this step that teachers will have to help children make connections between the two aspects. Their increased maturity, along with a more developed number sense, will make it easier to integrate the two concepts.

I feel that it is important that the ACME definition (2008: 4) be extended to include the 'quantity value' aspect rather than just focusing on the 'column value' aspect, and that the point be made that an understanding of the former precedes an understanding of the latter. This should prevent a 'limited' conception of place value from being enshrined in the final version of the Rose Review.

Ian Thompson is Visiting Professor at Edge Hill University, Ormskirk, Lancashire.

Addition (front-end method)

```
  47
 +76
 110
  13
 123
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Subtraction (by partitioning)

```
  500 + 60 + 3
 -200 + 40 + 1
  300 + 20 + 2
```

Subtraction (by complementary addition)

```
 326
-178
   2  (→180)
  20  (→200)
 100  (→300)
  26  (→326)
 148
```

Multiplication (by grid method)

×	20	7	
50	1000	350	1350
6	120	42	162
			1512

Division (by chunking)

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6)196
  -60   6×10
  136
  -60   6×10
   76
  -60   6×10
   16
  -12   6×2
    4     32
Answer:   32R4
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References

ACME (Advisory Committee on Mathematics Education) (2008) *Mathematics in Primary Years: A discussion paper for the Rose Review of the Primary Curriculum*, ACME www.acme-uk.org/downloaddoc.asp?id=101 (accessed 21/04/09)

DCSF (Department for Children Schools and Families) (2008) *The Independent Review of the Primary Curriculum: Interim Report (The Rose Review)*, London: DCSF http://publications.teachernet.gov.uk/eOrderingDownload/IPRC_Report.pdf (accessed 21/04/09)

DfEE (Department for Education and Employment) (1999) *Framework for teaching mathematics from Reception to Year 6*, DfEE, London

DfES (Department for Education and Skills) (2006) *Primary Framework for Literacy and Mathematics*, Norwich: DfES

Plunkett, S. (1979) Decomposition and all that rot, *Mathematics in School*, 8(3) 2 – 5

Ruthven, K. (1998). The use of mental, written and calculator strategies of numerical computation by upper-primary pupils within a 'calculator-aware' number curriculum. *British Educational Research Journal*, 24(1), 21–42

Thompson, I. and Bramald, R. (2002) *An investigation of the relationship between young children's understanding of place value and their competence at mental addition*. Final report submitted to the Nuffield Foundation. (Department of Education, University of Newcastle upon Tyne) www.ianthompson.pi.dsl.pipex.com (accessed 21/04/09)