On the right track? ..... Ian Thompson

As a long-term advocate of the Dutch approach to teaching mental calculation to young children using an empty number line I was delighted when the National Numeracy Strategy emphasised the important role that could be played by such a resource (Framework, p.29). However, I have never really been happy with the NNS approach to number tracks and lines, and it is only recently that I have been able to pin down what it is that bothered me (apart from the obviously premature introduction of 'complementary addition' and 'compensation' on the empty number line in Year 3!).

The first reference to the number track occurs in Reception, and right from the start the information provided appears to be somewhat confusing. On page 9 of the Supplement of examples: Reception we read that children should:
Begin to recognize 0 as the numeral associated with 'none', or the space before 1 on the number track.
Children are also asked to:

> Stand on 6 on the floor number track. Hop back to 0. Say the numbers as you go.

In addition, there is a fleeting glimpse of a portable floor number track that includes a zero in the introduction to the NNS video Mathematical activities for the Foundation Stage. And yet, on page 12 of the Framework we are shown a number track (or is it a washing line?) that begins at 1, where children are asked to put in the missing numbers. The Introduction to the Framework fails to make things clearer, arguing that 'For Reception and Year 1 number tracks with the spaces numbered to 20, rather than number lines with the points numbered are helpful'. The question that springs to mind is: 'Should number tracks include zero or should they not?' (I shall return to this question later).

A second, related issue that could be clearer in the early part of the Framework concerns the words cardinal and ordinal. The first mention of the word cardinal (and the only one I seem to be able to find) is in the context of 'zero':

> Begin to recognize 'zero' as the cardinal number associated with 'none'...

although the concept is implicit in an earlier outcome:

> Begin to realise through a variety of counting opportunities that the last number name spoken is the answer to 'how many' and tells you how many there are.

The word ordinal is first mentioned in the Framework on page 13:

> Begin to understand and use in practical contexts ordinal numbers to denote position: first, second, third, fourth...

However, I feel that this statement offers a limited conception of what ordinal numbers are: they don't just end in -th. For example, when I referred above to page 13, the 'thirteen' was being used in this context as an ordinal number. I was not referring to 13 pages, but to the specific page that follows 12 and precedes 14. Similarly, when in the process of counting - allocating number names in one-one correspondence to a collection of items - I am using the number names in an ordinal context until after the last number name is spoken, at which point there is an ordinal-cardinal switch: the number name allocated to the last object counted comes to represent the cardinal value of the collection i.e. it tells you how many items there are. The potential for confusion between cardinal and ordinal number is clearly illustrated in the following scenario. After counting a collection of, say, four objects a young child is asked 'show me three', and offers the third item counted rather than three discrete items. At one level this is a perfectly reasonable response, as the particular item offered by the child was actually 'named' as 'three'.

Combining the two issues raised, I would argue very strongly that, if children are expected to make sense of the number track (followed later by the number line and the empty number line), it is important that the track **begins at one rather than zero**. I say this because beginning at '1' ensures that cardinality is linked to ordinality: when you count up to '6' on the number track, i.e. allocate the number name 'six' to a specific space on the track (ordinality), this number also tells you how many spaces you have counted (cardinality). If the first space is designated '0' (zero), then this is obviously not the case: the numeral '6' is the seventh space! I feel that this is the strongest argument in favour of a 1 to 10 (or 1 to n) number

track (see Thompson, 2003a for a more extended argument in favour of the 1-100 square rather than the 0-99 version). 'Zero' can be introduced in 'counting back' activities as the place you land when you jump out of the number track from the space that contains the numeral '1', and, of course, '0' needs to be included when number lines are introduced.

It is important to be aware that calculating on a number track or number line is very different from calculating with objects or working things out mentally. For example, you cannot add 4 and 3 on a number line in the same way that you add them physically or mentally. The '4' and the '3' are positions on the line: they have no cardinality as they are ordinal numbers. So, addition on a number line has to be carried out in a different way: in this case by taking three steps forward from the '4'. To get a better feel for how number line (or track) calculations involve different strategies, work out 4 + 3 mentally using 'counting-on' and note the words that you say (aloud or to yourself). Then repeat the exercise using 'counting-on' on a number line. Make a mental note of any differences in the language used and the actions involved? Do this before you read on.

For the mental calculation you probably said Four... five, six, seven, (having kept track of the number of number names you said, i.e. three of them, five, six and seven). However, for the number line calculation you may well have put your finger on the '4'; said one, two, three as you moved your finger along the line; and then looked to see where your finger ended up (i.e. on the '7'). I would contend that, even though we describe both of these strategies as 'counting-on', they are conceptually very different procedures. I would also argue at this juncture that over-use of a fully-numbered number line could lead to children developing a simple algorithm for the addition and subtraction of 1- or 2-digit numbers. For example, to find 15+7 you simply place your finger on 15, count seven places to the right and read off the answer. To find 15-7 you place your finger on 15, count seven places to the left and read off the answer. Very simple and very efficient, in that all you have to be able to do is count and remember in which direction to count. Obviously, few number relationships are being learned, and even fewer useful mental calculation strategies are being developed during the execution of these procedures.

But now, fortunately, the NNS has produced a package that dispels most of my doubts and worries: a CD-ROM and charts entitled Using models and images to support mathematics teaching and learning in Years 1 to 3 (available to all teachers of Y1-3 from DfES publications phone 0845 60 222 60). Looking in particular at one of the six charts in the pack – Models and images for ordering numbers to 100 – we see that the use of 1-20 and 1-100 bead strings in conjunction with number lines of equivalent length and structure is highly recommended. This acknowledges that if children are going to make sense of the number line they have to relate the numbers on the line to actual quantities that they represent. There is also an example of a 1-100 number strip being cut in nine places to make a 1-100 square.

Also included on the CD-ROM are six Interactive Teaching Programs (ITPs)[1], several of which are used in the brief video sequences. Ordering Numbers is an animation where 10, 20 or 100 beads slide onto the screen in fives of two colours, and are then cleverly linked to a number line marked in fives, thereby stressing the important cardinal/ordinal connection. Another ITP, Difference, allows the user to set up two different-sized groups of beads; these then overlap, clearly showing the difference between the two groups. Two separate number lines repeat the sequence and an arrow 'jumps' from the smaller to the larger to illustrate the difference.

This emphasis on cardinality – or what is sometimes called the 'quantity' aspect of a digit or a number – is currently being seen in the literature as an important element in helping children to develop number sense (Grauberg, 1988); to use visualization to improve mental calculation skills (Dabell, 2003[2], Clausen-May, 2003), and to develop a more sophisticated understanding of place value (Thompson and Bramald, 2002[3], Thompson, 2003b). But it all starts in the early years by ensuring that children work with a 1-10 rather than a 0-9 number track to help them make the cardinal/ordinal connection!

REFERENCES

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Grauberg, E. (1988) Elementary Mathematics and Language Difficulties, Whurr Publishers, London.
Thompson, I. and Bramald, R. (2002). An investigation of the relationship between young children's understanding of the concept of place value and their competence at mental addition. (Report funded by the Nuffield Foundation). University of Newcastle upon Tyne.[3]
Thompson, I (2003a) 1 to 100 rules O.K.? Mathematics Teaching, 185 (in press).
Thompson, I (2003b) Place value: the English disease? in I. Thompson, Enhancing primary mathematics teaching, Open University Press: Maidenhead.

[1] The Progression charts and the ITPs can be downloaded from: www.standards.dfes.gov.uk/numeracy/publications/
[2] For a copy of Mecky Turner's report featured in Dabell's article contact: turmail@lineone.net
[3] If you would like a free copy of this report in pdf or Word form, e-mail ianthompson.pi@btopenworld.com

We've got mail!!

I receive lots of emails asking for, and giving, information but not too many that are commentaries on articles or on primary maths education in particular. If you'd like to start a topic, request help or information, or comment on something you've read, do drop me a line/email. I'd be happy to hear from you.

Dear Editor,
Wendy Singleton's article 'Mind the gap' (Volume 7, Issue 2) provided an extremely clear and succinct account of the National Numeracy Strategy's approach to written calculation – I shall definitely direct my PGCE students to it. However, anyone interested in thinking about the weaknesses and limitations of this approach might like to read the chapter, 'Deconstructing the National Numeracy Strategy's approach to calculation' in Ian Thompson (2003) Enhancing primary mathematics teaching, Open University Press.
Yours sincerely,
Ian Thompson

Ian's book has just been received for review. Do contact me if you'd like to receive a copy in return for a few hundred words.

I am investigating the use of the interactive whiteboards in mathematics at our primary school. Do you have contacts of schools where they are successfully using these whiteboards? I would like to contact them to arrange a visit or to discuss their potential so that I can support a "drive" for purchasing one for our school – if that is the best way forward.
Alternatively, or additionally, would you recommend any particular manufacturer or software for the boards?
I would appreciate any help or advice you could pass my way.
Many thanks, Mike Ferguson

I'm not sure where, geographically, Mike is to be found, but I'll pass on any information and publish any general comments readers would like to make.