Ian Thompson offers an insight into mental strategies, and suggests games to encourage mental calculation

Mind over mathematics

It is now a decade since the *Cockcroft Report*[1] commented on the decline in the emphasis given to the teaching of mental arithmetic in primary and secondary schools. Many of the commercial mathematics schemes of the 1960s and 1970s had begun to emphasise the importance of 'understanding' in the teaching and learning of mathematics. They had rightly dispensed with those pages of 'mental work' which characterised more traditional textbooks, and which were designed to be used as warm-up activities before beginning the 'mechanical' and 'problems' sections of the books.

Several writers have discussed the anxiety and stress that some adults confess to having experienced when doing mathematics at school. Many hated the undue emphasis on memory, accuracy and instantaneous recall associated with mental arithmetic. Fortunately (except for its annual resurrection in Key Stage 1 SATs!) this aspect of teaching and learning mathematics has become much less emphasised.

Mental arithmetic

In recent times a greater awareness of the difficulties that children experience with standard algorithms, combined with the increasing use of calculators by young children, has led to a resurgence of interest in mental methods. The Non-Statutory Guidance for mathematics National Curriculum informs us that:

'The heavy emphasis placed on teaching standard written methods for calculations in the past needs to be re-examined...the use of mental methods as a first resort in tackling calculations should be encouraged'.

In this context mental arithmetic is given a different emphasis. No longer are we talking about the instantaneous recall of known facts in a potentially threatening situation, but about the application of personal, albeit possibly more time-consuming, mental calculation methods. At issue is children's confidence in their ability to tackle problems in their head using their own idiosyncratic mental algorithms rather than resorting immediately to paper and pencil. In order for this confidence to develop children need to be helped to ascertain what these personal methods are; to share them with their teacher and classmates; to become engaged in a discussion of the various methods used by their peers, and to work in a secure and supportive environment.

Regrouping

In previous articles I have suggested activities which can help children develop the mental calculation strategies of counting-on[2], doubling[3] and using complements in ten[4]. This article concentrates on the specific strategy that I call 'regrouping'. At its simplest, 'regrouping' involves considering, say, 54 as 50 (fifty) and 4 (four), rather than as 5 in the tens column and 4 in the units column. This will be illustrated in some of the following examples. The ability to regroup depends on a confident understanding of the place value structure of our number system, and in the boxes are some activities which should help children develop this understanding.

Dominic wrote his explanation of the solution to 153 + 207 as follows:

'100 and 200 is 300
and 50 and 0 is 50
and 3 and 7 is 10
so that's 60 so it's 360'.

Jonathan (83 + 98) wrote:

'Two 100s are 200.
Take 30 take 10 is 170.
8 + 3 = 11.
170 + 11 =181'.

Scott (37 + 46) regrouped in a very idiosyncratic way:

'40 + 43 = 40 + 40 = 80 + 3 = 83'.

and Molly produced the most original adaptation of the basic strategy when calculating three lots of 72. Her explanation was:

'Three 50s are 150.
Six 10s are 60 and three 2s are 6...
so it's 226' (one of the few errors!).

Some children used the regrouping strategy for subtraction. Blake (54 – 27) gave the following oral explanation of his strategy:

'I have 54 and 20 and then I took away and you are left with 34...
and then you take the 4 away and you are left with 30...
and then you take the 3 away and you are left with 27'.

Twenty-seven has been regrouped as 20 + 4 + 3 in order to facilitate the subtraction process. Had the original problem been 55 – 27 I have no doubt that Blake would have treated 27 as 20 + 5 + 2.

Other children used the regrouping strategy for multiplication. Jane (four lots of 144) simply wrote:

'400 + 160 + 16 = 576'.

showing that she had treated 144 as 100 + 40 + 4 and worked from left to right finding four lots of each group.

The most ingenious bit of regrouping came from Andrew, who was only seven. He was asked to find four lots of eight, and gave the following oral explanation for his correct answer (you may have to read his answer twice!):

'Three sevens are 21. Add on all the next ones to get 8. You have three more units so you get 24. Add on 8 to get 32'.

When asked how he added the final 8 he replied:

'24 and 6 makes 30, and 2 more makes 32'.

What these children have in common is the fact that they are at all times working meaningfully with methods and strategies that are under their control. They are all adopting an approach which involves working with quantities rather than working with symbols, and they are therefore able to operate on the numbers involved with understanding instead of by rote.

The collection of games and activities – some old, some new, some borrowed – can be used to develop children's understanding of our place value system, and help them gain sufficient confidence to be able to create their own ways of operating with multi-digit numbers.

Exploding a number

This activity – illustrated in the 'explosion' of 374 in Figure 1 – has been found to be suitable for use with the whole class as a blackboard activity, with small groups working together or with individual children working at their own level. The children have to find the greatest number of solutions that will total the number in the centre.

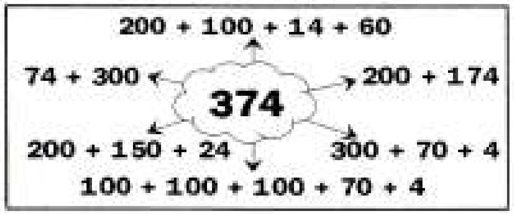


Figure 1

Fill the box

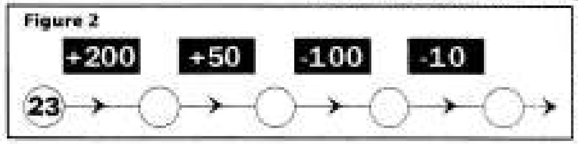
This activity requires the production of a variety of worksheets using the layout illustrated in Figure 2.

Aim
By adding or subtracting multiples of ten or a hundred (or even a thousand) to given numbers, the children experience the changes that this produces in the digits comprising these numbers.

Equipment
One calculator for each child, with worksheets at the appropriate level of difficulty.

Procedure
In this activity (figure 2) the children follow the arrows and use their calculators to help them add the numbers in the rectangles to the numbers in the previous circles. They should make note of what happens to the individual digits comprising the numbers in the circles.

Figure 2



Modifications
1 The activity outlined above is really an introduction to the more difficult but more useful problem of filling in the rectangles when given the numbers in all the circles.
2 Alternative worksheets leaving out the addition and subtraction signs makes the activity a little more difficult.
3 Blank grids can be provided for the children to create similar problems for their friends to tackle.

Delete-a-digit

This is a pacifist version of 'Space Invaders'.

Aim
The game builds on the previous activity, and consolidates, by providing immediate feedback, the basic idea that in a number such as 438 the 3 means 30 or three tens.

Equipment
One calculator for each pair of children.

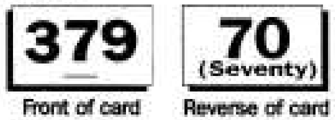
Rules
Player A enters a three-digit number in the calculator, and player B has to delete the digits by making three subtractions. The digits must be removed one at a time by working from left to right. The numbers subtracted will be either multiples of a hundred, multiples of ten or single-digit numbers.

Modifications
1 Less-able or younger children can work with two-digit numbers and more-able or older children can work with four- or five-digit numbers.
2 The game can be made more complex by having the children delete the digits in ascending order, so that in a number like 537 the three would be removed first by subtracting 30, the five would be deleted by subtracting 500, and the seven would be removed last of all. In this version of the game children have to distinguish between the size of a digit and the size of its place value meaning.

Turnover

Equipment
You need a set of 24 home-made cards (for 2, 3, 4, or 6 players) with two- and three-digit numbers on one side and with one of the digits in the number underlined. The value of this underlined digit is written on the reverse of the card, thereby rendering the activity self-correcting (see Figure 3).

Figure 3



In the example given, 70 has been written as 'seventy' rather than as 'seven tens'. This is because children, thinking of their 'hundreds, tens and units' work, can sometimes give the answer 'seven tens' but fail to make the important connection that this is actually equal to 'seventy'.

Rules
The pile of cards is placed in the middle of a small group of children. Player A picks up a card and has to state the value of the underlined digit. The card is then turned over. A correct answer means that the child keeps the card, and an incorrect answer results in the card's being placed at the bottom of the pile. This activity is repeated by all the players until there are no cards left. The winner is the child with the greatest number of cards at the end of the game.

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References
1 *Mathematics Counts: the Report of the Committee of Enquiry into the Teaching of Mathematics in Schools* HMSO 1982.
2 'Making games that count' Ian Thompson *Child Education* June 1992.
3 'Double up and double back' Ian Thompson *Child Education* August 1990.
4 'Mind games' Ian Thompson *Child Education* December 1989.