Mind games

Ian Thompson

Do we put enough emphasis on mental calculation? Ian Thompson examines how children solve problems in their heads, and suggests ways of helping them develop strategies

It is probably true to say that in most primary schools there is less emphasis on the development of mental arithmetic skills than there used to be. Yet in the world of work and leisure the majority of arithmetical problems encountered are solved in one's head by means of a variety of idiosyncratic methods.

The National Curriculum Council document *Mathematics in the National Curriculum* puts a great deal of emphasis on the need for children to possess highly developed skills in mental calculation, and several of the attainment targets for 'Number' contain references to these important, but often neglected, skills.

When calculators are freely available for children to use, one of the most important skills needed at every level is the ability to do a quick mental check to make sure the answer shown in the calculator display is reasonable.

There is obviously a need for a reappraisal of what is meant by 'mental arithmetic' in the late 1980s — a time when calculators are used in school, at home, at work — and for school homework!

Methods children use

Whilst carrying out some research I tape-recorded and analysed individual discussions with 59 top infants and 44 first-year juniors from three different schools (two rural and one inner-city) on the methods they use to solve simple, and occasionally not so simple, addition, subtraction and multiplication problems in their heads.

The children were told that they could work out the problems in any way they liked, and that I was not interested in whether they got the answer right or wrong but in the methods they had used to find their answer.

The questions were of the type: What does three and four make? Having ascertained that all the children were reasonably conversant with addition, subtraction and multiplication symbols, I placed a card containing the same problem in symbolic form, i.e. 3+4, in front of each child in turn.

A wide range of interesting strategies was revealed, and the most striking aspect was the creativity and ingenuity shown by the children in adapting concepts or techniques learned in one context for use in another.

The rest of this article will concentrate on one particular strategy used by different individuals in different types of problem situation. The mathematical idea underlying this chosen strategy is 'complements in ten'. For example, 7 is the complement in ten of 3, and 1 is the complement of 9. This is covered in most mathematics schemes by the 'story of ten' type of activity, but none of the schemes offers more than one or two activities. The concept is not usually treated as the important building block for mental arithmetic that it would appear to be.

'Complements in ten'

The most common use of this 'complements in ten' or 'sums to ten' strategy was in the context of addition. When Mark was asked to explain his correct answer to 7+8 he replied: 'I had the 8 and I knew that 8 and 2 was 10 ... then because 7 is an odd number it's got 5 more (!) ... 10 and 5 is 15.'

Other children used a similar technique:

Blake (8+5): '8 and 2 is 10, and then you've got 3 left, and when you add another 3 on you've got 13.'

Richard (7+4): 'I knew what 7 and 3 is, so I added 1 more.'

Anna (7+4): 'Well there'll be 3 more to make 10, and then add 1 on.'

Scott (8+6): 'If 8 is 2 less than 10, add 2 off the 6, and take all the leftovers ... so you just put them to 14.'

All of these children are able to ascertain what is needed to build one of the numbers up to ten. They split the other number into two appropriate parts, and then add these two parts separately by making use of their knowledge of the effect of adding a single-digit number on to ten.

The most striking aspect was children's ingenuity in adapting techniques for other uses.

Other children used the strategy for subtraction:

James (12–4): 'I knew that if you take 2 away that's 10, and you've got another 2 left, and you take away that and it's 8.'

Tim (15–9): 'I took away 5 from 15 then I took away 4 ... that's 6.'

David (12–4): 'Because I knew that 10 (he meant 12) take 2 is 10 and take away another 2 is 8.'

None of these children was taught this strategy, and yet when you analyse the technique you realise that it is quite subtle.

The units digit of the larger number tells you how many to take away first in order to reduce it to ten. You then have to take away the same amount from the smaller number, and the answer tells you what you still have to take away from the ten. The answer is the complement in ten of this number. Imagine trying to teach this technique formally!

The technique was also used for more difficult subtractions:

Mark (24–7): 'When I've got 24 and I take away 4 it makes 20, and I know that 3 and 17 makes 20 so I have 3 left to make 17.'

And for multiplication:

Andrew (3 fours): '2 fours are 8 ... add 2 is 10 ... add another 2 is 12.'

Stephen (4 sixes): '6 and 6 is 12 and another 6 is 18. When you add on another 2 it makes 20 and you've got 4 left, and so it's 24.'

The following games and activities may help other young children to develop similar strategies for use in a variety of mental arithmetic contexts.

One of the most important skills at every level is the ability to do a quick mental check.

Guess it

This is a simple calculator game for children who are just learning their 'complements in ten'.

Children play in pairs. The first player enters a number from 1 to 9, and her partner has to add a number to it so that the calculator display reads 10. If the answer is incorrect the calculator is passed to the first player who tries to make the display read 10 (by adding or subtracting, depending on the error made), and thereby gains a bonus point.

For more able children the display is not cleared, and play continues by making the number up to the next multiple of 10. This idea can be extended up to 100.

Familiar games with a twist

In the next four games — all adaptations of traditional games — you need two or three sets of cards showing the numerals 1 to 9. An optional extra is a calculator for checking the accuracy of calls. All the games are for two to four children.

Snap

Shuffle and deal the cards. Play as for normal 'Snap', but call when two successive cards sum to 10.

The winner is the player who captures all the cards.

Pelmanism

Using the same cards, shuffle them and spread them face down in a rectangular array. Each player in turn selects two cards to turn over without altering their position. If the two cards sum to 10 the player keeps the 'trick' and has another turn; otherwise the cards are returned to their original position, and the players try to remember where they are for future use.

The player with the most tricks at the end wins.

Dominoes

Make a set of dominoes, without the doubles, using the numerals 2 to 8 (or 3 to 7 if you want a smaller set). Play as for normal dominoes except that adjacent dominoes must sum to 10.

Rummy

Use as many sets of 0 to 9 cards as there are players. Deal four (or more) cards to each player, and put the rest face down in one pile with the top card face up to form a 'rejects' pile next to it.

At the start of the game players put down pairs of cards which sum to 10 — their tricks. They next take turns to select a card from the top of either pile, and then place one of their cards on the 'rejects' pile. The winner is the player with the most tricks when all the cards have been picked up.

Extensions

All of these games can be adapted or extended in the following ways:

— add 0 and 10 cards or 1 and 9 dominoes;

— use cards or dominoes with numbers in word form;

— combine numerals and words;

— develop 'sums to 20' by using 1 to 19 cards and dominoes.

The first resort

As well as practising these mental arithmetic skills, children need to realise that they can use them to deal with more difficult problems, like 7+5 or 18+6. But they also need, in the words of the Mathematics Working Group, to come to 'regard mental methods as a first resort when a calculation is needed'.

Ian Thompson is a lecturer in mathematics education at the University of Newcastle upon Tyne.