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# MENTAL CALCULATION STRATEGIES FOR ADDITION AND SUBTRACTION PART 2

by Ian Thompson

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## Introduction

Several official documents have recently been published which exhort primary teachers to place more emphasis on the development of mental calculation strategies when teaching mathematics. An important teaching objective at this level is that children should be able to recall or calculate quickly all the basic number facts within this range, and it is strongly believed that emphasizing these strategies can help achieve this objective.

Part 1 of this article looked at the most common methods used by young children for the addition and subtraction of numbers to twenty, and concluded that, because of their usefulness in later two-digit work, the key strategies were:

- bridging up and down through ten (using complements in ten);
- partitioning single digit numbers (7 is  $5+2$  or  $4+3$  ...);
- compensation (adding 9 by adding 10 and subtracting 1).

It is important to realize that a big difference between working with one-digit and two-digit calculations is that children are generally not expected to learn two-digit number facts by heart, but rather to develop general strategies which will enable them to add or subtract mentally any pair of two-digit numbers: strategies upon which written algorithms for three-digit numbers can be built at a later date.

If teachers are to be successful in teaching mental strategies then they need, as a minimum requirement, to be familiar with the different methods that children use, and have appropriate teaching strategies for developing and extending these procedures. In this article I shall discuss some recent research evidence concerning mental addition and subtraction strategies for two-digit number calculations.

## Mental Calculation Strategies with Numbers from 20 to 100

Thompson and Smith (1999) suggest that there are four main two-digit addition and subtraction strategies used by children for mental calculation, and these are partitioning, sequencing, a hybrid method and compensation. There is

also a fifth strategy, particularly useful for difference problems, which in England is known as 'complementary addition'.

### Partitioning

In England the most common strategy used is *partitioning*, so-called because the numbers to be added or subtracted are both partitioned into multiples of ten and ones. This strategy is sometimes called the '*split*' method, and in research literature is known as the 1010 (ten-ten) procedure.

Sammy ( $63+56$ ):

*'119 ... I added the sixty and fifty first and then added three and six.'*

Rebecca ( $68-32$ ):

*'Thirty-six ... I took away thirty from sixty and then took away two from eight.'*

Sammy has split the 63 into 60 and 3; has split the 56 into 50 and 6; has added the 60 and the 50 together; has added the 3 and the 6 together; and has then added the two sub-totals together (110 and 9) to get the correct answer 119. Rebecca has split the 68 and the 32 in a similar way; has calculated two separate subtractions and then combined the two answers to get 36.

One problem with the basic partitioning strategy is that it breaks down with subtractions which necessitate regrouping, and this is one reason why in The Netherlands they teach a different strategy first. However, some pupils get round this difficulty by employing a clever procedure ... illustrated by Emily who is tackling 86-39:

*'47 ... I took away the 30 from the 80 which makes 50, and then I took away six from nine ... and then I took away the three which makes forty-seven.'*

Even though the language used by Emily is potentially confusing, there is no doubt whatsoever that she understands this sophisticated manoeuvre. She appears to be making the 'smaller from larger' error, but it is more likely that she is thinking along the lines of 'I've still got three more to take away'. What I am fairly certain about is that she is *not*

thinking '6 take 9 is minus 3' – although her strategy can obviously be explained mathematically in this way. This 'negative numbers' interpretation is another example of an adult interpretation of a child's method: the adult explains the procedure by referring to the underlying mathematics whereas the child executes the calculation via discrete steps of an appropriate procedure. Several other children – generally the more able – used Emily's procedure.

### Sequencing

The *sequencing* method is used less frequently by children in this country, although it is the method taught to children in The Netherlands.

Paul (55+42):

*'97 ... I added the 40 to the 55 and that made 95 ... and another two is 97.'*

Sarah (54-27):

*'27 ... I knew that 54 take away 20 is 34 and then you need to take another 7 off ... well I took the 7 ... I did it by because I know 3 and 4 equals 7 because the 4 made it down to 30 then you just have to take another 3 off.'*

Paul has split the 42 into 40 and 2; has added the 40 onto the 55 to get 95; and has then added on the two to get the correct answer 97. Sarah has calculated 54-20; partitioned the remaining 7 into a 4 and a 3 (because of the 4 in the 34); subtracted the 4 from the 34 to give her a handy multiple of ten; and then used her knowledge of complements in ten to subtract the remaining 3 from the 30 (since 10-3 is 7 then 30-3 is 27). Notice the use of 'bridging' and 'partitioning one-digit numbers' in Sarah's calculation.

The most important difference between this strategy and partitioning is that, in the case of sequencing, one of the numbers in the calculation is retained as a whole, and chunks of the other number are added to or subtracted from it. Unfortunately, the National Numeracy Strategy *Framework for Teaching Mathematics* (DfEE, 1999a) fails to distinguish adequately between these two important and conceptually different procedures, and the QCA's guidance on teaching mental calculation (QCA, 1999) actually classifies five different strategies under the 'partitioning' heading – even one that involves no partitioning at all.

Alternative descriptors for the sequencing strategy are the *jump* or *cumulative* method, and in the research literature it is known as *N10* (start from the number and add multiples of ten). It is described as a *jump* strategy because it can be easily represented – practically or mentally – on a number line, where the procedure starts at one of the numbers and progresses towards the answer in jumps along the line where conveniently-sized chunks of the second number are added or subtracted (see Paul and Sarah's transcripts above). This strategy copes effectively with 'awkward' subtractions by making use of the 'bridging through ten' procedure to deal with the difficult part of the calculation.

### Mixed method

This hybrid strategy combines the first stages of the partitioning method with the later stages of sequencing strategy.

Nicholas (37+45):

*'82 ... I added the 40 and the 30 which made 70 ... and then added the 5 which made 75 ... and then added the 7 which made 82.'*

Laura (68-32):

*'36 ... I knew 60 take away 30 is 30 and add the 8 on is 38 ... and then you take the 2 from the 8 which is 36.'*

Both children start by splitting the two numbers and then finding the sum (or the difference) as they would if using the partitioning strategy. They then sequentially combine each units digit separately with their interim totals. Nicholas adds the 5 and then 7, whereas Laura adds the 8 and then *subtracts* the 2. The basic partitioning strategy needs to be modified in order to deal with subtractions like 85-37, but the mixed method does not, as the following example shows.

Afzal (54-27):

*'27 ... I took 50 from ... I took 20 from 50 ... which gave me 30 ... then I added the 4 and took away the 7.'*

Afzal's calculation procedure shows excellent number sense. He has extended Laura's method to deal with difficult subtractions. He has subtracted the multiples of 10 to get 30 and has then added back the 4 to get 34 before subtracting the 7. Further discussion revealed that the 7 was subtracted by splitting it into 4 and 3, removing the 4 to get back to 30 and then using complements in 10 (or, more precisely, in 30) to get 27. This method is also known as the *split-jump* method, the *cumulo-partial* or *10S* (partitioning followed by sequencing).

### Compensation

Compensation is a strategy which involves adding or subtracting a number larger than the number specified in the calculation – usually the next higher multiple of 10 – and then modifying the answer by 'compensating' for the extra bit added or subtracted.

Lauren (46+39):

*'85 ... Well, I said that was 40 so 40 plus 46 is 86 and you've got to take 1 away.'*

Sarah (86-39):

*'47 ... I added the 39 up to 40 ... I took 40 away from 86 is 46 ... and then added another 1 ... Well, I knew that the units wouldn't change ... it would just be the 10's ... and 8 take away 4 is 4.'*

Both girls have treated 39 as 40. Lauren has then added the 40 to the 46 and then taken 1 away, whereas Sarah has subtracted it from 86 and then added 1 on. This would appear to demand a high level of confidence with numbers and a good implicit understanding of the inverse relationship between operations. The user of this strategy needs to be aware that adding 1 more than required must be compensated for by taking 1 away from the answer. In a similar way, subtracting one more than necessary implies adding 1 on to the answer.

Compensation is a very efficient strategy, but it is not used as spontaneously by young children as it is by adults. However, this does not mean that children cannot be taught to become efficient exponents of the procedure. It can be introduced in a structured way on the empty number line, where a 'jump too far' and the 'compensatory jump' in the opposite direction are easily represented. It is often used when operating with numbers ending in 9, but if taught

systematically, as it is in The Netherlands, it can be used successfully for two-digit numbers ending in 8 or even 7.

Abigail (37-18):

*'19 ... I added 2 onto the 18 which made it 20 ... I took 20 from 37 which equalled 17 ... I added the 2 back on which equalled 19'.*

Other descriptors used for this strategy are *over-jump* and *N10C* (start with the number, add the next multiple of 10 and then compensate).

### Complementary addition

There is a further subtraction algorithm which the English call 'complementary addition' or 'shopkeeper arithmetic', and which the Dutch call 'adding to 10' (*A10*). In fact, some maths educators do not consider this strategy to be a subtraction strategy at all: where is the subtraction if you solve the problem by addition? This procedure is a particularly powerful one for difference problems such as: 'John is 107 cm tall and his father is 162 cm. How much taller is the father?'. This type of problem almost demands that you start at 107 and add on chunks until you get to 162.

Complementary addition, like compensation, is also used more by adults than by children. However, this strategy is successfully taught to children in some European countries, where work on difference problems is used to stimulate its use. The introduction of the strategy is more effective if, initially, the problems involve numbers that are particularly close together.

James (73-68):

*'Five ... I added 2 onto the 68 ... so then that made 70, and then I added another 3 on'.*

James has noticed the proximity of the two numbers and has had the confidence, despite the subtraction sign, to use the simpler procedure of finding the difference using complementary addition. For numbers with a larger gap it is often difficult to keep a tally of the numbers added on without writing something down (hence the power of the empty number line for supporting this particular mental strategy, where children keep track of their calculation informally on paper).

### Conclusion

One of the aims of Session 6 of the National Numeracy Strategy's *Three-day Course: Tutor's Pack* (DfEE, 1999b) is to 'show how written calculations build upon the mental strategies taught' (p.63). If we really do accept this principle of building on and extending children's own informal mental calculation strategies then teachers are going to need a substantial amount of support and guidance to help them achieve this aim. This article has covered only the most basic level of this support, namely, the provision of information to help make teachers aware of the most common mental calculation strategies used by young children for two-digit addition and subtraction.

The next step would be for teachers to gain experience of working with these strategies so that they can recognize their similarities and differences. They need to discuss them with their pupils in order to appreciate their strengths and weaknesses and become able to recognize them in their children's explanations. They also need: support to develop teaching strategies for helping children who are struggling with two-digit calculation methods; help in extending those

children whose methods might need refining; suggestions of ways of introducing the different strategies and sequencing learning activities; and guidance to help them develop written algorithms from their children's mental strategies.

However, we have little or no experience of *teaching* mental calculation in this country and a conspicuous dearth of publications recommending suitable tried and tested teaching strategies and learning sequences. The National Numeracy Strategy documents and related publications give the impression of having all the answers, but this is, unfortunately, not the case: recently published documents such as the *Framework for Teaching Mathematics* (DfEE, 1999a), the *Three-day Course Tutor's Pack* (DfEE, 1999b) and QCA's *Teaching Mental Calculation Strategies: Guidance for Teachers at Key Stages 1 and 2* (QCA, 1999) provide insufficient support for helping teachers develop the necessary skills and expertise [see Thompson (under review) for a more detailed critique]. ☒

### References

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*Just a thought:*

**TAN 2000 – would  
you do it  
in degrees?**

**What's the difference  
if you used radians?**

**How close an answer  
can you get if you  
knew the values  
for the 'tan' of  
360°, 180° and 60°?**