IS THE NATIONAL NUMERACY STRATEGY EVIDENCE-BASED?

Ian Thompson

Introduction

The National Numeracy Strategy – the most focused attempt ever made to raise the standards of numeracy in English primary schools – was launched amidst much euphoria in 1999. In the Foreword of *Numeracy matters* [1], the preliminary report of the Numeracy Task Force, David Reynolds explains that a concerted effort was made to 'deliberately set out to be evidence-based rather than adopt any particular 'faiths' about numeracy...', and to produce a set of recommendations which reflected such an approach. This claim is reinforced in the final report, *The implementation of the National Numeracy Strategy* [2] which stresses that one of the key beliefs guiding the Task Force was the importance of 'looking at the evidence...to find solutions to any problems with mathematics attainment'.

The framework for teaching mathematics from reception to year 6, [3] plays a vital role in making this theoretical underpinning accessible to teachers, and constitutes the major source of information on the way in which the Task Force has interpreted this 'evidence'. The *Framework* states explicitly that the approach taken to the teaching of mathematics is based on four key principles: dedicated maths lessons every day; direct teaching; controlled differentiation; and an emphasis on mental calculation. This last principle is developed further in the introduction to the document, where it states quite clearly that 'oral and mental work will feature strongly in each lesson'.

Hughes [4] has questioned the theoretical basis for the National Numeracy Strategy, and Brown et al. [5] have reviewed some of the research literature which underpins it. This article focuses more specifically on the approach to the teaching of mental addition and subtraction as set out in the *Framework*, and seeks to ascertain the extent to which its recommendations are based on research evidence.

Key Stage 1: numbers to 20

One noticeable feature of the Reception section of the *Framework* is the shift in emphasis concerning the teaching of early number. Gone are the traditional sorting, ordering and matching ('pre-number') activities found in most commercial mathematics schemes currently in use. This approach has been replaced by a detailed set of structured recommendations for helping children acquire the complex skill of counting. Amongst these recommendations are suggestions that children should be taught to:

- recite the number names in order;
- continue the count from a given number;
- stop counting at a given number;
- count back from a given number;
- count up to a given number;
- count groups of objects in a variety of different ways.

This focus on 'recitation' and 'enumeration' is clearly influenced by the work of Schaeffer [6], and the major emphasis given to the development of the wide-ranging sub-skills needed in order to count effectively is based on the work of Gelman and Gallistel [7] and Fuson [8].

In the recommendations for the remaining years of Key Stage 1, mental work is extended by building on the skills acquired in Reception and developing them into a range of mental calculation strategies for adding and subtracting single-digit numbers. Included in these strategies are:

- counting on and counting back;
- using near doubles;
- using fives (partition into 5 and a bit);
- adding or subtracting 10;
- bridging up or down through 10;
- adding or subtracting 9 by working with 10 and compensating;
- counting up (for difference problems).

For more details on these strategies see [9].

The structure and sequence elaborated in this section of the *Framework* for teaching basic addition and subtraction strategies with numbers to 20 progresses from the development of counting strategies through to the recall of number bonds and the derivation of new facts. This approach is clearly based on the work of Carpenter and Moser [10] who identified these strategies in a longitudinal study in the United States mapping children's development from Grade 1 to Grade 3 (Y2 to Y4).

Key Stage 2: numbers from 20 to 100

Research findings

Much less is known about children's mental calculation strategies for adding and subtracting two-digit numbers. However, Thompson and Smith [11] have recently reported the results of a project which involved interviewing 144 children in Years 4 and 5 (Grades 3 and 4) about their two-digit mental addition and subtraction strategies. Their findings suggest that there are four main strategies used by children for mental calculation with numbers from 20 to 100. These strategies are: partitioning, sequencing, a method which is a mixture of the two and compensation. There is also a fifth strategy, generally used for difference situations, which in Great Britain is known as 'complementary addition'.

Partitioning

The most common strategy used by children in the sample was partitioning, so-called because the numbers to be added or subtracted are both partitioned into multiples of tens and ones. Sarah, finding 37+45, explained:

'82...I added the thirty and the forty which is 70 and then I added the nine and the six which is 15...and then I added them together'

She has split the 37 into 30 and 7; has split the 45 into 40 and 5; has added the 30 and the 40 together; has added the 7 and the 5 together; and has then added the two sub-totals together (70 and 12) to get the correct answer 82.

Sequencing

The sequencing strategy is used less frequently by children in this country, although Dutch children are formally taught this method. Paul was asked to explain his answer to 37+45, and said:

'Eighty-two...I made that just forty and added forty onto 37...That's 77...and then added the five so that was 82'

Notice that Paul has kept the 37 intact; has split the 45 into 40 and 5; has added the 40 to the 37 giving 77; and has added the remaining five to obtain 82. It is important to observe that with the sequencing strategy only one of the numbers is split.

Mixed method

Some children used a combination of the two strategies described above, and this method is clearly illustrated in Nasrin's answer to 37+45:

'82...I just...forty add thirty is 70...and then five and that made 75...and add the 7 comes to 82'

She has first partitioned both numbers and found the sum of the multiples of ten; (40+30=70). She has 'put back' the 5 that she originally removed from the 45 to make 75, and has then added on the remaining 7. Some children would add the seven first to make it easier for themselves.

Compensation

Compensation is a strategy which involves adding or subtracting a number larger than the calculation demands – usually the next multiple of ten – and then modifying the answer by 'compensating' for the extra bit added or subtracted. Lauren gave a succinct explanation of her method for calculating 86−39:

'47...86 minus 40 and then added one'

Her response shows that she has realised that subtracting 40 is easier than subtracting 39 – so long as she remembers to compensate at the end.

Complementary addition

An alternative name for this algorithm is 'shopkeeper arithmetic', dating from the days when shopkeepers used to count out the change into your hand! It is a particularly powerful strategy for solving a range of difference problems. James's method for finding 73−68 illustrates the method:

'Five...I added two onto the 68...so then that made 70, and then I added another three on'

It is likely that the proximity of the two numbers stimulated James to find the difference by addition. For numbers with a larger gap it is more difficult to keep a tally of the chunks added on, and so children should be encouraged to make jottings to support their calculations. For more details on these strategies see [12].

Strategies for 2-digit calculation in the framework document

The mental strategies that children should be taught in Years 4, 5 and 6 are listed as 'outcomes' in section 6 of the *Framework* (pp. 40-47), and those objectives considered to be central to pupils' achievements are called key objectives. They are highlighted in bold type, and teachers are advised to give top priority to these in their planning. One particular key objective, 'Use known number facts and place value to add and subtract mentally any pair of two-digit whole numbers' covers several pages and lists a range of outcome skills that pupils should learn and practise. In the following discussion this particular objective has been taken as the umbrella term to cover the more specific strategies which will be addressed individually. I have reorganised these into two discrete categories to facilitate analysis:

Group A

- partition into tens and units, adding the tens first;
- add or subtract the nearest multiple of ten and then adjust;
- find a small difference by counting up.

Group B

- identify near doubles, using known doubles;
- count on or back in repeated steps of 1, 10, 100 or 1000;
- use the relationship between addition and subtraction.

In discussing these objectives I shall argue that those strategies listed in Group B are less important at this level than those in Group A, and consequently should require much less emphasis in teachers' planning. All the examples quoted are taken from the relevant 'outcomes' section of the *Framework* for Year 4.

Partition into tens and units adding the tens first

The use of the word 'partition' in this objective suggests a possible link with the research evidence discussed above. The two examples given in the text to illustrate this strategy are set out next to each other in the following way:

24+58=82 because it is 20+50=70 and 4+8=12, making 70+12=82;

98−43=98−40−3 which is 58−3=55

One problem with this example, given that there are no explanatory notes, is that the two most important mental calculation strategies have been linked together under the same heading: the addition example illustrates the *partitioning* strategy where the subtraction exemplifies *sequencing*. As the underpinning logic of the two procedures is different, children whose natural inclination is towards one method could well experience difficulty if teachers concentrate their teaching on the other.

It is unfortunate that the *Framework* does not stress the important difference between the two methods: with the partitioning strategy both numbers are split into multiples of ten and ones before the calculation is effected, whereas with sequencing strategies one of the numbers remains 'non-partitioned' during the calculation whilst chunks of the other number are added to or subtracted from it. This may seem an academic point, but it is just as important as understanding the difference between the two standard algorithms for written subtraction, 'decomposition' and 'equal additions'.

Another important implication of this difference is that sequencing methods lend themselves naturally to representation on an empty number line, whereas partitioning methods do not. An increasing number of official documents [13, 14, 15, 16, 3] and commercial publications make references to teaching children about the empty number line, but none of them make reference to this point. Children who use the more common partitioning strategy are likely to have great difficulty when asked by a teacher to demonstrate their method to the rest of the class on such a line [see 17].

Add or subtract the nearest multiple of ten and then adjust

There is a link between this objective and the compensation procedure described in the research section above. The strategy is introduced in the *Framework* in Year 1 as 'Add or subtract 9, 19, 29...or 11, 21, 31...by adding 10, 20, 30...and adjust by 1'. In Year 1 children add 9 to a single-digit number; in Year 2 they add 19 to, and subtract 9 from, a two-digit number; and in Year 3 they should be able to add or subtract 9 to or from a three-digit number and work out 78−49 as 78−50+1. In Year 4 pupils are expected to be able to work out mentally 74+58 and 128−67 using this method.

This strategy is referred to by researchers as 'compensation'. However, this word is only used in the *Framework* to describe a specific pencil and paper procedure (Y123, p.45 and Y456, pp. 48-51). This is unfortunate because what is effectively the same strategy in mental and in written form has been allocated a different descriptor in separate parts of the document. This situation has the potential to cause confusion, particularly for those teachers who are trying to make links between written methods and children's personal mental strategies.

Another possible source of confusion in this section is the inclusion of the addition or subtraction of 11, 21 and 31 within this objective. Adding 21 to a number should be done in exactly the same way as adding 22, 23, 24 or 25: by using either the partitioning or sequencing strategies described above. In the research literature the term 'compensation' is specifically used to describe the procedure for adding or subtracting numbers which are close to a *larger* multiple of ten.

There is also potentially confusing information in *Teaching Mental Calculation Strategies* [16, p.30] which includes 'compensation' as one of the five 'partitioning' strategy. Unfortunately, however, compensation is the only two-digit mental calculation strategy which involves no partitioning at all. It is even more unfortunate that the QCA document then proceeds to describe the strategy in the following way:

The number to be added is rounded to a multiple of 10 plus a small number or a multiple of 10 minus a small number.

The normal meaning of 'rounding' involves giving an answer to the nearest 10 (or 100...), not 'to the nearest 10 plus or minus a small number'. Once a number has been 'rounded' the difference between the original number and the rounded one is normally ignored and this replacement number is then used for further calculation or given as the answer.

Find a difference by counting up

The examples given in this section are 92−89, 403−386 and 4000−3993: all subtractions with small differences. In fact, in the relevant section of the 'Yearly Teaching Programmes' (p.18) this objective is actually listed as 'Find a *small* difference by counting up'. The strategy is really a simplified version of the more general 'complementary addition' procedure, adapted to deal with numbers that are close together. However, complementary addition is more than just a strategy to be used for small differences: it is a powerful alternative procedure that can be used for a wide range of subtraction situations regardless of the size of the numbers. It is particularly useful for dealing with contextualised difference problems, and is taught in this way in the Netherlands, where it is called 'adding to ten'. Also, by including the words 'counting up' in this objective the *Framework* stresses the low-level skill of counting to find the solution rather than the higher-level skill of calculating. 'Adding up to' would have been a more relevant phrase to use at this level of work, so that children are '*adding up* to the next 10, and then up to the next 100'.

The three objectives discussed above do show some links with the strategies found in the research literature, although there are some important differences and some idiosyncrasies. However, a similar connection is more difficult to establish for the following three objectives.

Identify near doubles, using known doubles

This strategy is extremely powerful for children who are still building confidence with numbers to twenty. Knowing the doubles facts and being aware of how to make use of them gives children access to other number facts that they have not yet learned. However, the importance of this strategy as an appropriate or useful method for working with two-digit numbers is questionable. How many people would find 48+46 by using the (known!) fact that double 47 is 84? It would seem sensible to suggest that it is more important for children to learn generalisable strategies so that they can deal mentally with any pair of two-digit numbers, rather than use a more specific strategy that is dependent upon their knowledge of the doubles of the two-digit numbers. Incidentally, given any pair of two-digit numbers to add or subtract, the relevant mathematics would suggest that there is only a 1 in 45 chance of their being near doubles!

Count on or back in repeated steps of 1, 10, 100 or 1000

One of the examples in this section recommends that children work out 643+50 by counting on in tens from 643. However, by this stage of their mental calculation work children should be able to separate the 600 and the 43 and use the argument that since 43+50 is 93 (a skill acquired in Year 2), then the answer must be 693. A similar argument obtains for 387−50, which the document suggests should be done by counting back in tens from 387. Another example, 2003−8=1995, is to be done by counting back in ones. This would appear to be a much lower level procedure than the bridging strategy that children were introduced to in Year 1, and which was further developed in Year 2. I would expect children to tackle this calculation by first subtracting 3, as they would with 23−8, and then extend their knowledge of 'subtracting a single digit from a multiple of 100' (Year 3) to help them work out 2000−5.

Using the relationship between addition and subtraction

It is important for children to realise that if they know that 8+7=15 then they also know what 7+8, 15−7 and 15−8 are. This is very useful information for children working with numbers up to twenty, where an important aim is to ensure that they are ultimately able to recall number facts quickly. However, the example given in the *Framework*:

'Continue to recognise that knowing one of:
36+19=55, 19+36=55, 55−19=36, 55−36=19
means that you also know the other 3'

would appear to be much less useful – particularly for me, as I don't really *know* any of them!

Conclusion

The first part of the mental calculation section of the Framework, extending roughly to the end of Key Stage 1, is firmly based on research evidence. However, the mental methods included for children in Years 4, 5 and 6 appear to be based on the same early years research, where the recommended strategies are simply logical extensions of those from Key Stage 1. Research carried out since the Framework was written suggests that different strategies need to be emphasised and developed [11, 12].

Teachers are going to need some support over and above that currently provided by the Framework and related training materials [18] if they are going to extend their own understanding of two-digit mental calculation strategies to a satisfactory level. This understanding is an essential first step towards the successful development of the mental methods used by their children. Combined with the provision of practical suggestions for teaching mental calculation and an increase in teachers' confidence, this should lead to an improvement in children's informal and formal written procedures.

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Ian Thompson works at the Department of Education, University of Newcastle upon Tyne.

TWENTY-FIVE YEARS AGO

From MT71, page 4

The following are 'thoughts in progress'. I don't apologise for that, since no lecture has any right to do anything else, but I admit I wish the progress were more spectacular and the story accompanied by many more detailed illustrations. There is, indeed, a very great deal still to do. . .

It might as well be said at once that one relatively minor way to humanise mathematical education would be to stop forcing mathematics on children who have had ample opportunity to discover that it means nothing to them – at least, unless we really have some techniques that can bypass their failure and astonish them into success; but also, hopefully, arrange to make it accessible to them if and when they subsequently decide that they were mistaken.

Humanising mathematical education is not to be confused with encouraging mathematics teachers to come on like warm accepting therapists. No doubt there are occasions when this is what they have to be, though probably not as many as some teachers would like to believe. Respect for children is generally more useful to them than affectionate hand-holding; it may often need to be tough and abrasive too. But respect is not enough, either, since it doesn't necessarily carry any insight on the teacher's part into what to do, and it doesn't ensure the presence of skills he ought to have in order to carry out what needs to be done if children are to be helped to learn.

In the context of teaching, affection is incomplete without respect, and respect incomplete without insight and skill. Perhaps we should remind ourselves that the teacher-learner relationship is far too complex to be describable in single statements. It seems as if we frequently manage to isolate some important ingredient of teaching or of learning and then make the cardinal mistake of elevating the part so that we begin to take it for the whole. The sentimental and mechanistic simplifications of the teacher's role that result when the more demanding pedagogical skills and expertise are ignored or denied provide sad examples.

David Wheeler

This extract from MT71 was selected by David Lingard, and is part of the transcript of a lecture at the 1975 ATM Easter Conference. 'I still, 25 years on, give this article to many of my students, and it seems to me as relevant today as it was then – perhaps more so in the current climate.'