DIVISION BY 'COMPLEMENTARY MULTIPLICATION'

by Ian Thompson

We are all very familiar with 'complementary addition' as a procedure for dealing with comparison/difference situations or, at a more general level, for tackling any particular problem involving subtraction. The procedure can be used either as a mental calculation strategy or as a written algorithm, although, unfortunately to my mind, the *Framework* uses different nomenclature for the procedure when used in these two different contexts, choosing to describe the mental version as 'Find the difference by counting up through the next 10, 100 or 1000', and reserving 'complementary addition' for the written version. This has led to many teachers not recognizing the two strategies as one and the same. Another problem with the *Framework* lies in the fact that the examples offered always involve numbers that are fairly close together; in fact, the strategy is described in Key Stage 1 as 'Find a small difference by counting up'. This, of course, ignores the fact that the mental or written algorithm can be used successfully with numbers of any size.

The complementary addition procedure is one that we use quite often in real-life situations: giving change for a five pound note when an item has been bought for £2.83; calculating how long we have to wait for the 11:26 ferry if the time is 09:48; working out how old someone is who was born in 1938... For this, and other reasons, I feel very strongly that if schools decide to opt for the teaching of one specific written subtraction algorithm – usually a choice between decomposition, equal additions or the use of negative numbers – they should think seriously about complementary addition, particularly as this is the only written procedure for subtraction that progresses smoothly from mental methods, through jottings (usually on the empty number line) to a more formal written algorithm (Thompson, 2003a).

In this article I consider the strategy that is currently recommended in the *Framework* for teaching division, and then make a case for what I have chosen to call the 'complementary multiplication' algorithm. I think most teachers would agree that the standard algorithm for division is difficult to teach. If we look at the Y6 example given in the *Framework* (977 ÷ 36) and use the standard algorithm to work it out, the language we might use for this calculation would probably begin: *36 into 97 goes twice; 72 from 97 is 25; bring down the 7, making 257; 36 into 257...* I have discussed elsewhere (Thompson, 2003b) the difficulties involved in attempting to use the more meaningful language recommended in National Numeracy Strategy materials. Many children, trainees (and some teachers) have experienced difficulty in correctly calculating by long division, and this has led to the recommendation of what has become known as the 'chunking' method – considered by many to be a new method. On looking through some of my older mathematics education books, the earliest example of the repeated subtraction division algorithm that I can find is in the Schools Council's 1965 Curriculum Bulletin No. 1, *Mathematics in Primary Schools*. However, there is reputed to be a slightly older example of its use in the Kahun papyrus, circa 1825 BC!

Division by Chunking

The language and thinking associated with the calculation of 977 ÷ 36 using chunking (Fig. 1) would be something like the following: *I need to find out how many 36s there are in 977. I know that there are ten of them in 360, so there are twenty of them in 720. If I take this 720 from 977 I'm left with 257. Five 36s are 180 (half of 360), and 180 from 257 is 77. Two 36s are 72, so there are going to be 5 left over. So, the answer is 27 (20 + 5 + 2) remainder 5.*

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36 )977
    -720      36 × 20
     257
    -180      36 ×  5
      77
     -72      36 ×  2
       5
                 27 remainder 5
```

Fig. 1

The educational reasons given for the teaching of this algorithm appear to be sound: the method does not demand that children follow a prescribed set of steps in a specific order; the least able children can find the answer by subtracting small chunks, whereas the more confident can subtract larger chunks; because the children are in control of the size of the chunk they choose to subtract, the algorithm provides a level of differentiation that is not possible with the standard algorithm. The examples shown in Figure 2 – which progress from the least to the most compact – illustrate a range of approaches to the calculation 977 ÷ 36 using chunking:

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977        977        977         977         977         977
-36        -72   2   -360   10   -360   10   -720   20   -720   20
941        905        617         617         257         257
-36        -72   2   -360   10   -360   10   -180    5   -252    7
905        833        257         257          77           5
-36        -72   2    -36    1   -180    5    -72    2
869        761        221          77           5
-36   1    -72   2    -36    1    -72    2
833        689        185           5
    27r5       27r5        27r5        27r5        27r5        27r5
```

Fig. 2

The more sophisticated the strategy (i.e. the larger the chunks) the fewer subtractions are needed. However, a particular problem with the argument that one of the strengths of the procedure lies in the fact that children can remove chunks of any size that they choose is that the least confident children, when subtracting small chunks, as in the first example above, actually make 27 subtractions. This, of course, provides 27 opportunities for making a subtraction error, whereas the more confident children only make two, three or four subtractions. It is also likely to be the case that those children performing many subtractions are the very children who have difficulties with subtraction.

Given that subtraction is more difficult than addition, it would seem sensible to try to develop an algorithm for division that depends on addition and multiplication; this is the focus of the remainder of this article. Obviously, a great deal of preparatory work needs to be done to develop the pre-requisite skills listed below and to build up children's confidence and competence with calculations involving smaller numbers. These pre-requisite skills are doubling, halving, multiplying by 10, accurate addition and a good sense of the relative size of numbers. Two different methods of introducing complementary multiplication are suggested below.

Division by Complementary Multiplication

Method 1

In the context of 977/36, this method involves the generation of a section of the 36 times table using doubling, multiplying by ten and halving – all skills that children should have acquired by this stage (Fig. 3).

1	36
2	72
4	144
10	360
5	180

Fig. 3

The actual calculation procedure involves building up to 977 by adding selected chunks to an appropriate starting number. As with the subtractive chunking method, the size of chunk will be a function of the ability or confidence of each child, but in this case children will be using the less difficult operation of addition. Using the table in Figure 3, one way to proceed might be as follows:

360	10	
360	10	
720		*That's twenty 36s, but it's still smaller than 977.*
~~360~~	~~10~~	
~~1080~~		*Too big! Cross out 360 and 10. Try five 36s.*
720		
180	5	
900		*This is only 77 short.*
72	2	
972		*'Only 5 short.'*
	27	*'So the answer is 27 remainder 5.*

Fig. 4

(Remainders are found by complementary addition, with children always ensuring that the remainder is never larger than the divisor.)

Other ways that children might solve this problem using complementary multiplication are shown in Figure 5.

A slightly different approach, using the same method, would be to add twenty 36s and forty 36s to the table in Figure 4 by multiplying 72 and 144 by 10. This should help children to see that the answer is going to lie between 20 and 40, and may well spur them on to start building up from 720.

72	2	360	10	720	20
72	2	180	5	144	4
144		540		864	
72	2	180	5	72	2
216		720		936	
72	2	180	5	36	
288		900		972	
72	2	36			
360		936			27 r 5
72	2	36			
432		972			
72	2				
			27 r 5		
		27 r 5			

Fig. 5

Method 2

This method involves a little more 'trial and improvement' than Method 1, but may be more relevant for children who have had experience of Egyptian multiplication (Fig. 6).

An example of Egyptian multiplication

To calculate 34 × 26, use doubling to generate parts of the 34 times table:

1	34
2	68
4	136
8	272
16	544

As 26 = 16 + 8 + 2, cross out the products not required, and add the rest:

~~1~~	~~34~~
2	68
~~4~~	~~136~~
8	272
16	544
	884

So, 34 × 26 = 884

Fig. 6

With division, the idea is to set out an Egyptian 'doubling' table for the divisor (Fig. 6), stopping only when the dividend is exceeded. For 977 ÷ 36 this would proceed as follows:

1	36
2	72
4	144
8	288
16	576
32	152

Fig. 7

As before, there are several different ways of approaching the problem using this strategy. The table in Figure 7 tells us that the answer is somewhere between 16 and 32, and that it is nearer to the latter than the former. We could use the table to calculate three 36s by adding the results for one and two, giving 108. Multiplying this by 10 shows that 30 × 36 is also a bit too big. Alternatively, as we know that the answer lies between 16 and 32, we could start building up from 576 (36 × 16) (Fig. 8).

Those children who have experience of Egyptian multiplication should be able to understand the underlying principles of Method 2. However, in this procedure the

576	16	
288	8	
864		
~~144~~	4	
~~1008~~		*Too big. Cross out 144 and 4. Try two 36s.*
864		
72	2	
936		*Try another 36.*
36		
972		*'Only 5 short.'*
	27	*'So the answer is 27 remainder 5.'*

Fig. 8

numbers are usually a little more unwieldy, as multiples of ten are not involved. The strength of Method 1 lies in the fact that the required table is easier to generate, involving just two doubles, one multiplication by ten and one halving. If necessary, larger values can be found by simply multiplying existing values by ten. There is no doubt in my mind that complementary multiplication would be easier for children to learn than chunking. What is now needed is a research project to test this hypothesis.

References

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Keywords: Division; Chunking; Multiplication; Algorithms; Written calculation.

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