DECONSTRUCTING CALCULATION METHODS PART 2: SUBTRACTION

In the second of a series of four articles, Ian Thompson deconstructs the primary national strategy's approach to written subtraction. The first in the series, on addition, was published in *MT202*.

The approach to subtraction, as set out on pages 8 to 11 of the primary national strategy's (PNS) 'guidance paper' Calculation (DfES, 2007), is divided into three stages: using the empty number line; partitioning; and expanded layout leading to column methods (called 'standard methods' in the consultation document).

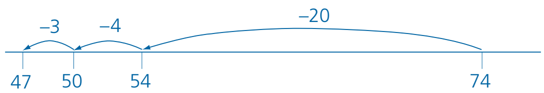
Stage 1 Using the empty number line

The recommendations in this stage parallel those outlined for addition, and four examples – including the following – are provided:

74 − 27 = 47 worked by counting back (*figure 1*).

I would argue that the *counting back* descriptor in this example is a misnomer. Most of the literature on early calculation methods suggests that counting back involves reciting backwards as many number names as the number you are subtracting, and then giving the last number that you said as your answer to the subtraction. An example from the original framework (DfEE, 1999a) supports this interpretation.

Figure 1



We made six mince pies. We ate two of them.
How many pies are left?
(Count back two from six: 5, 4. Say together '6 take away 2 is 4')

Section 4, page 17.

What the strategy illustrated in *Figure 1* actually involves are the following procedures:

- subtracting a multiple of ten (74 − 20 = 54);
- partitioning the ones in such a way that 50 will be reached after the next subtraction (7=4+3);
- subtracting the requisite number of ones to reach 50 (54 − 4);
- and then subtracting the remaining ones (50 − 3 = 47).

No counting back is involved.

Also included in stage 1 is a sub-section entitled *The counting-up method*. This involves adding appropriate chunks to the smaller number until you reach the larger one. The calculation can be recorded on an empty number line or in columns. This strategy allows the number of steps to be reduced as the children's mental strategies improve. The examples in *figures 2* and *3* illustrate this point.

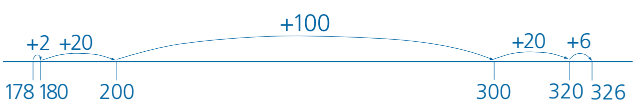


Figure 2

```
  326
− 178
    2  (→180)
   20  (→200)
  100  (→300)
   20  (→320)
    6  (→326)
  148
```

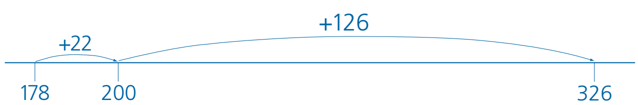


Figure 3

```
  326
− 178
   22  (→200)
  126  (→326)
  148
```

It is interesting that in the original framework (DfEE, 1999a) children were expected to:

> *Understand subtraction as:*
> - *taking away*
> - *finding the difference between*
> - *complementary addition*
>
> Section 5, page 29.

However, in the new version only the first two bullet points are mentioned: complementary addition has been dropped. This is odd, as this is precisely the strategy recommended for the *counting up* method. Incidentally, it is good to see that the PNS correctly distinguishes here between counting on and counting up – which is more than they did in a different publication (see Thompson, 2006).

Stage 2 Partition

This section seems to be a needless and irrelevant backward step. The last example in stage 1 uses the *counting up* method to solve a quite difficult calculation: 22.4 − 17.8. Here the document recommends using basic partitioning to calculate 74 − 27, even though we have just seen it used several times in the six *counting up* examples provided in stage 1. Also, the empty number line used to illustrate the calculation is exactly the same as that in stage 1 (see *Figure 1* above). The real purpose behind this section is obviously to emphasise that decomposition – the strategy covered in the next section – is the favoured strategy of the PNS. This is also confirmed by the fact that counting up, the only other written method mentioned, is consigned to stage 1, accompanied by a note saying:

> *The counting up method can be a useful alternative for children whose progress is slow…*
>
> Calculation (1) page 8.

Stage 3 Expanded layout leading to column method

As was mentioned in the first of this series of articles, in order to appease the 'anti-standard-algorithms' group that responded in a somewhat negative manner to the consultation document, the PNS simply substituted the word 'column' for 'standard', without changing any of the actual algorithms. The main subtraction goal in this document is to have children progress inexorably towards the standard compact decomposition method. The procedure is introduced via the 'expanded method', initially using an example that requires no 'exchanging' or 'borrowing'. We are taken through a progressive sequence from 563 − 241 (*figure 4*).

500 + 60 + 3		563
−200 + 40 + 1		241
300 + 20 + 2	leading to	322

Figure 4

culminating in 563 − 278 (*figure 5*).

				150 400 50 13		4 15 13
500+60+3	or	400+150+13	or	~~500~~+~~60~~+~~3~~	leading to	~~5~~ ~~6~~ ~~3~~
−200+70+8		−200+ 70+ 8		−200+70+8		−2 8 7
		200+ 80+ 5		200+80+5		2 8 5

Figure 5

To someone who can already perform written subtraction, this progression no doubt appears perfectly logical. However, we know from research and the combined experience of many teachers that children have great difficulty with the decomposition algorithm. Hart (1989) found that children struggled to make the anticipated connections between the manipulation of practical apparatus and their pencil and paper calculations when learning the algorithm.

One particular weakness of the layout suggested here leads to a specific error made by children: they add rather than subtract one or more of the partitioned elements. For example,

500 + 60 + 3	
−200 + 40 + 1	
300 + 20 + 2	is erroneously calculated as:
500 + 60 + 3	
−200 + 40 + 1	
300 + 20 + 4	or even
500 + 60 + 3	
−200 + 40 + 1	
300 + 100 + 4	

Figure 6

This type of error is, of course, a function of the recommended layout, which incorporates addition symbols between the separate partitions in a context where children are expected to subtract. This could be avoided by relating the layout to place value cards, where 563 looks like 500 60 3 when the three components are separated. The resulting calculation would be written as

500	60	3		500	60	3
−200	40	1		−200	−40	−1
300	20	2	or	300	20	2

Figure 7

Another problem with this algorithm is that children are expected to be able to make non-canonical partitions, such as 73 = 60 + 13 or 563 = 400 + 150 + 13. Ross (1989) has shown that children generally find this difficult.

Given that a clear aim of the PNS is to develop written procedures that build on children's mental strategies, it is important to point out that in the extensive literature on children's idiosyncratic mental calculation strategies there is, to my knowledge, no example of any child inventing decomposition. This would appear to provide important evidence that might help explain why children find decomposition so difficult. On the other hand, *counting up* is the only subtraction procedure with a built-in natural progression from basic mental strategy through a range of levels of jottings and informal notation to a more formal written notation.

One reason for teaching this procedure rather than decomposition is its widespread use in real-life situations, such as giving change (hence its alternative descriptor, *shopkeeper arithmetic*); calculating elapsed time; finding the difference between two given measurements, etc. Another reason is the fact that children can choose the size of the chunks that they choose to add on and the number of steps they take to complete the task. This can range from the five steps of *figure 2* to the two steps of *figure 3*. In ***MT202*** Ian Sugarman wrote about an alternative subtraction algorithm that he had trialled with a group of children. Like the counting up or complementary addition algorithm this procedure also offers children a certain amount of choice as to how they tackle each calculation (Sugarman, 2007).

It is a great pity that in the early stages of its development the national numeracy strategy did not set up a research project to attempt to ascertain which of the wide range of written algorithms incorporated into the framework were the most 'child-friendly'.

Ian Thompson is visiting professor at Northumbria University.

References

DfEE (Department for Education and Employment) (1999a) *Framework for Teaching Mathematics from Reception to Year 6,* DfEE

DfES (2007) *Guidance paper–Calculation* www.standards.dfes.gov.uk/primaryframeworks/downloads/PDF/101Guidance_CalculationFinal.pdf (accessed June 2007)

Hart, K.M. (1989) Place value: subtraction, in D. Johnson (ed.) *Children's Mathematical Frameworks 8 – 13*, NFER-Nelson

Ross, S. (1989) Parts, wholes and place value: a developmental view, *Arithmetic Teacher*, 36(6): 47-51

Sugarman, I. (2007) The same difference, *MT202*, May, 16-18, ATM

Thompson, I. (2006) *The revised framework: a document to count on?* www.atm.org.uk/professional-services/opinion/ATM-Opinion-Thompson PNS-Revision-2006-Sep.pdf

Thompson, I. (2007) Deconstructing calculation methods: Part one, *MT202*, May, 14-15, ATM