DECONSTRUCTING CALCULATION METHODS, PART 3: MULTIPLICATION

In the third of a series of four articles, Ian Thompson deconstructs the primary national strategy's approach to written multiplication. The first two articles in this series were published in *MT202* and *MT204*.

The approach to multiplication, as set out on pages 12 to 15 of the primary national strategy's 'Guidance paper' ***Calculation*** (DfES, 2007), is divided into six stages: mental multiplication using partitioning; the grid method; expanded short multiplication; short multiplication (by Y4); two-digit by two-digit products (by Y5); and three-digit by two-digit products (by Y6).

Stage 1: Mental multiplication using partitioning

The document recommends various types of 'informal recording' for mental multiplications. (Isn't there something oximoronic about 'recommended' informal recording? Surely, the phrase implies that the child jots down anything that helps her keep track of her calculation or that offers her support during the calculation process?). It is suggested that children might record a mental calculation such as 14×3 as:

$$14 \times 3 = (10 + 4) \times 3$$
$$= (10 \times 3) + (4 \times 3) = 30 + 12 = 42$$

I would argue that this procedure is far too formal. It is a fact that some children in Years 3 and 4 have an implicit understanding that multiplication is distributive over addition. For example, Thompson (1993) showed 13 children's different informal written methods for tackling a problem that could be solved either by adding four 144s or calculating 144×4. The four children who solved the problem using multiplication all showed an implicit understanding of distributivity (see Andrew's solution in *figure 1*).

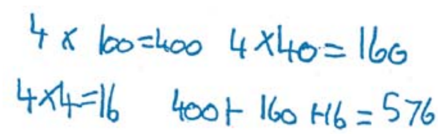


Figure 1

However, I doubt whether any of the children would have been able to express their calculation as:

$144 \times 4 = (100 + 40 + 4) \times 4 =$
$(100 \times 4) + (40 \times 4) + (4 \times 4)$

Just as young children develop an implicit awareness that addition is commutative without being able to articulate this verbally or on paper, older children similarly become aware of the distributivity of multiplication over addition. Expressing these laws of arithmetic formally in words or in writing is too difficult for most young children, and is a particularly redundant exercise in this case, given that we are dealing with mental calculation strategies.

Stages 2 & 3: The grid method and expanded short multiplication

Figure 2 illustrates the recommended layout for using the 'grid' method for the calculation 38×7.

×	7
30	210
8	56
	266

Figure 2

I have never been particularly impressed by the strategy's notation for the grid method, given that different publications draw the grids in different and sometimes non-user-friendly formats. For example, compare this notation with that of DfEE (2000). Personally, I prefer to start with squared paper, where (in this case) the calculation would involve finding the number of squares in a 38 by 7 rectangle. The 38 could be partitioned in various ways: (10 + 10 + 10 + 8, 20 + 10 + 8 or 30 + 8), thereby allowing children control over the size of the smaller internal rectangles they would be working with. As the children gain confidence with the concepts underlying the procedure, they can progress to abbreviating by sketching rectangles that are no longer to scale. These sketches retain the partitioning and distributive aspects of the calculation, and can provide a mental model for more formal methods introduced at a later stage. (For more details see Thompson, 1996 or 1999.)

Two intermediate procedures (both unnecessary to my mind) link *figure 2* to the stage 3 algorithm shown in *figure 3*.

```
  38
 × 7
----
 210
  56
----
 266
----
```

Figure 3

This is described as 'expanded short multiplication', and involves working from left to right with quantities, ie finding 30 × 7 first. This written strategy develops quite naturally from mental multiplication methods and from the grid procedure.

I have argued elsewhere, with detailed examples (Thompson, 2002), that the aspect of place value underpinning mental calculation methods and informal written procedures is different from that which underpins the standard (or 'column') written algorithms: the former methods involve 'quantity value' (where 56 is interpreted as *fifty plus six*), whereas the latter procedures involve 'column value' (where 56 is interpreted as *five in the tens column and six in the ones column*). This primary national strategy (PNS) document does not appear to acknowledge that such a difference exists.

Stage 4: Short multiplication

Here 'the recording is reduced further, with carry digits recorded below the line' (DfES, 2007, p13). The document gives the impression that there is a smooth transition between the algorithms illustrated in *figures 3* and *4*.

```
  38
 × 7
----
 266
----
  5
```

Figure 4

In order to perform the algorithm in *figure 3* correctly, we were informed that 'the first step in 38 × 7 is 'thirty multiplied by seven', not 'three times seven'; ie, working from left to right with quantities. However, for the algorithm in *figure 4* we are told 'The step here involves adding 210 and 50 mentally, with only the 5 in the 50 recorded.' What puzzles me is whether or not I am still supposed to be working from left to right. If this is the case, then surely, having mentally worked out the 210, followed by the 56, I would want to write down the 6 so that I don't forget it whilst I'm in the process of adding the 210 and the 50 as recommended. There seems to be little point in writing down the 5 (except that we have always done this with the standard algorithm!).

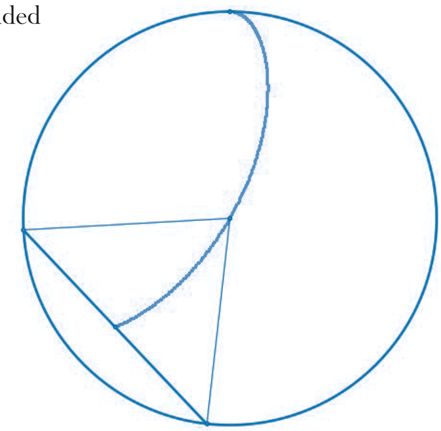
On the other hand, if I am actually calculating from right to left, then I need to work out the 8 × 7 first. In this case, wouldn't it be more sensible to write the 50 somewhere whilst I am writing the 6 in the ones column and working out 30 × 7. Doing this will help me remember that I have to add 50 to the answer to 30 × 7. If I write down 5, as suggested, am I not likely to add just 5 rather than 50 to my 210? You have to remember that up to this point in the development of the PNS approach to multiplication no mention has been made of working with numbers in columns. All the algorithms covered so far (plus those to be found in stages 5 and 6) involve working with quantities, not digits with specific column values.

Given the user-friendliness of the procedure in *figure 3* and its relation to mental multiplication and the grid method, why do we need to introduce the potentially confusing algorithm for 'short multiplication' at all? Yet again, it would appear to be a throwback to the past: we have always taught children 'short multiplication' before progressing to 'long multiplication'.

Stages 5 & 6: 2-digit by 2-digit products and 3-digit by 2-digit products

As stage 6 merely extends the recommendations in stage 5 to more challenging calculations, I shall confine my discussion to the latter.

Here, the grid method is extended to calculations like 56 × 27 (*figure 5 overpage*).



×	20	7	
50	1000	350	1350
6	120	42	162
			1512
			1

Figure 5

After introducing the basic grid layout, the document develops it into that shown in *figure 6*.

	50	6	
×	20	7	
	1000	350	1350
	120	42	162
			1512
			1

Figure 6

My opinion is that this layout has now lost any real relationship to the concept underlying grid multiplication. For example, what connection do the diagonal moves to find 6 × 20 and 50 × 7 have with the grid structure? Obviously, the aim is to prepare children for the final two algorithms in this stage (*figures 7 & 8*).

56	
× 27	
1000	50 × 20 = 1000
120	6 × 20 = 120
350	50 x 7 = 350
42	6 x 7 = 42
1512	
1	

Figure 7

56	
× 27	
1120	56 × 20
392	56 × 7
1512	
1	

Figure 8

The algorithm in *figure 7* retains the partial products calculated in the grid method (*figure 5*). However, no advice is given as to how to calculate the two separate products shown in *figure 8*. I am fairly sure that the vast majority of Y5 children would have great difficulty in working out 56 × 7 mentally. As mentioned, stage 6 deals with the multiplication of three-digit by two-digit numbers. The reader might like to ascertain the level of difficulty of the recommended algorithm by calculating 456 × 78 using this method.

In the first article in this series, I mentioned the brouhaha created when an earlier version of the PNS guidance paper proposed that all children should be using traditional standard methods of calculation for the four basic operations by the time they left primary school. Interestingly, the original document (and the current one, given that the changes were only to the language and not the content) does not actually recommend what most people would recognise as the standard algorithm for long multiplication. For example, 'standard algorithm language' for the calculation 56 × 27 would be something like "6 × 7 is 42. Put down the 2 and carry the 4. Next put down a zero. 5 × 7 is 35..." As we shall see in the final article in this series, exactly the same applies to the standard long division algorithm. Perhaps some of the negative publicity would have been avoided if the PNS team had acknowledged (realised?) this in the first place!

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References

DfEE (2000) *Five-day course: Course handbook*, p96, DfEE

DfES (2007) *Calculation* (PNS Guidance Paper) www.standards.dfes.gov.uk/primaryframeworks/mathematics/Papers/Calculation/ Accessed October 2007

Thompson, I. (1993) Thirteen ways to solve a problem, *Mathematics Teaching,* 144, 27-30

Thompson, I. (1996) User-friendly calculation algorithms, *Mathematics in School,* 25, 5, 42-45

Thompson, I. (1999) Written methods of calculation, in I. Thompson (ed), *Issues in Teaching Numeracy in Primary Schools*, Open University Press, 169-183

Thompson, I. and Bramald, R. (2002) *An investigation of the relationship between young children's understanding of the concept of place value and their competence at mental addition* (Report for the Nuffield Foundation), University of Newcastle upon Tyne Available to download at www.ianthompson.pi.dsl.pipex.com

Thompson, I. (2007) Deconstructing calculation methods Part 1: Addition, *Mathematics Teaching, 202*, May, 14-15

Thompson, I. (2007) Deconstructing calculation methods Part 2: Subtraction, *Mathematics Teaching, 204*, September, 6-8