DECONSTRUCTING CALCULATION METHODS, PART 4: DIVISION

In the final article of a series of four, Ian Thompson deconstructs the primary national strategy's approach to written division. The other three articles in the series appeared in *MT202*, *MT204* and *MT206*.

In the introductory section to division on page 16 of the primary national strategy's 'Guidance paper' Calculation (DfES, 2007), we are informed that:

> *'These notes show the stages in building up to long division through Years 4 to 6 – first long division TU ÷ U, extending to HTU ÷ U, then HTU ÷ TU, and then short division HTU ÷ U'.*

This progression seems to me to be somewhat 'logically-challenged': why long division before short division? However, the actual five-part sequence of stages set out on subsequent pages is slightly more logical (although still questionable): mental division using partition; short division of TU ÷ U; expanded method for HTU ÷ U; short division of HTU ÷ U; and long division.

Stage 1: Mental division using partition

It is recommended that children record mental division as follows:

$$\begin{aligned} 81 \div 3 &= (60 + 21) \div 3 \\ &= (60 \div 3) + (21 \div 3) \\ &= 20 + 7 \\ &= 27 \end{aligned}$$

I see several conceptual difficulties with this suggestion, over and above the inappropriate formality of the notation, which is, after all, recording for a *mental* calculation. The main problem concerns the distributivity of division. Multiplication is both left- and right-distributive over addition (and subtraction), whereas division is only right-distributive. This means that 7 × 12 can be calculated as (7 × 4) + (7 × 8) or (3 × 12) + (4 × 12), where either number is partitioned.

However, although 84 ÷ 7 is equivalent to (70 ÷ 7) + (14 ÷ 7), it is not equivalent to (84 ÷ 4) + (84 ÷ 3). This suggests opportunities for children to make inappropriate partitionings: they need to remember that only the dividend and not the divisor can be partitioned.

The second problem with this algorithm is that children are expected to be able to make 'non-canonical partitions', such as 73 = 60 + 13 or 563 = 400 + 150 + 13. Ross (1989) has shown that children generally find this difficult. Also, if the wrong partition is made, the result can be somewhat 'messy'.

$$\begin{aligned} 81 \div 3 &= (50 + 31) \div 3 \\ &= (50 \div 3) + (31 \div 3) \\ &= 16\tfrac{2}{3} + 10\tfrac{1}{3} \\ &= 27 \end{aligned}$$

Anghileri (2001) shows the workings of a child calculating 1256 ÷ 6 by finding, separately, 1000 ÷ 6, 200 ÷ 6, 50 ÷ 6 and 6 ÷ 6. These work out, respectively, to 106 r 2, 21 r 2, 8 r 2 and 1, and are then added to give 136 r 6.

Stage 2: Short division of TU ÷ U

I have written elsewhere (Thompson, 2003) about different interpretations of short division – both as a concept and as an algorithm. This guidance paper states that the short division method is to be recorded as:

$$\begin{array}{r} 20 + 7 \\ 3\overline{)60 + 21} \end{array}$$

This is then to be shortened to:

$$\begin{array}{r} 27 \\ 3\overline{)8\,{}^{2}1} \end{array}$$

We are informed that:

'The carry digit '2' represents the 2 tens that have been exchanged for 20 ones.' (DfES, 2007: 18).

Interestingly, this is the first time that the concept of 'exchanging' has been mentioned in the entire document, despite the fact that there is a detailed section on subtraction. Written subtraction algorithms are often based on a model that involves base-ten materials and the exchange of 'flats' for 'longs' (hundreds for tens) and 'longs' for 'ones'. The subtraction method recommended in this document, however, is based on the concept of 're-partitioning' (although the word is never actually used). Given the difficulties that children experience with exchanging in subtraction, even when preparatory work has been done with base-ten materials (Hart, 1989), it is difficult to believe that the recommended progression will be any more successful.

The progression from the third calculation to the fourth calculation also involves a shift from the quantity value aspect of place value (80 and 1) to the column value aspect (8 tens and 1 one) discussed in earlier articles in this series (Thompson, 2007 and 2008). However, this particular shift is slightly more complicated, in that children have to be able to partition 81 into 60 + 21, as illustrated in the third calculation, and then switch to interpreting 81 as 6 tens and 21 ones. Given the research mentioned above (Ross, 1989) that children have great difficulty making 'non-canonical partitions' such as 81 = 60 + 21, it would be informative to ascertain how easy they find the idea of 81 being equivalent to 6 tens and 21 ones.

Stage 3: 'Expanded' method for HTU ÷ U

The recommended method is the one we have come to know as 'chunking', where multiples of the divisor are subtracted from the number to be divided (the dividend).

6)196	
−60	6 × 10
136	
−60	6 × 10
76	
−60	6 × 10
16	
−12	6 × 2
4	32
Answer:	32r4

Having done some preliminary work on the development of estimation strategies, children are expected to progress to the following, more succinct notation:

6)196	
−180	6 × 30
16	
−12	6 × 2
4	32
Answer:	32r4

The way in which the strategy is presented in the document gives the impression that it comes in just two forms: you either repeatedly subtract the smallest multiple of ten of the divisor or you subtract the largest. In fact, the strength of the chunking algorithm lies in its great potential for differentiation: it allows for a range of levels of sophistication in children's confidence and understanding, in that the less confident can remove small chunks; the more confident can take away larger chunks; and the most confident can subtract the maximum-sized chunks. A more detailed analysis of this strategy can be found in Thompson (2005).

Stage 4: Short division of HTU ÷ U

In addition to the issues raised above, concerning the introduction of short division in stage 2, the following question is offered: Why do we need to teach a conceptually difficult strategy for dividing a three-digit number by a single-digit number when the chunking method introduced in stage 3 for solving three-digit by two-digit divisions is much easier to understand, allows for differentiation and is probably more effective?

Stage 5: Long division

The recommended method in this section involves estimating to find the maximum amount to subtract initially, as in stage 3, and then continuing with the standard chunking procedure.

	24)560	
20	−480	24 × 20
	80	
3	72	24 × 3
	8	
Answer:		23r8

One reason offered for the rather strange positioning of the 20 and the 3 down the left-hand side is to keep the links with 'chunking' – although I would have thought that the notation down the right-hand side and the calculation procedure itself might do that! A second reason given is that it reduces the errors that tend to occur with the positioning of the first digit of the quotient. As the first digit of the quotient is 2, written as 20 on both the right- and the left-hand side, I have difficulty in understanding what this talk of 'positioning' is all about. Also, as the answer is to be written at the bottom of the procedure, the extra inclusion of the 20 and the 3 down the left-hand side seems to be adding another level of potential confusion.

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    23
24)560
  −480
    80
   −72
     8
Answer:   23r8
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The document then tries to argue that the notation illustrated above is, in effect, the standard long division method. However, the language and thinking associated with the chunking method runs something like:

I need to find out how many 24s there are in 560. I know that there are ten of them in 240 and so there are twenty of them in 480. If I take this from 560 I get 80. Two 24s are 48, so four 24s are 96 – but this is too big. 48 plus 24 is 72, and 72 from 80 leaves 8. So, the answer is 20 and 3, that's 23, remainder 8.

On the other hand, the procedure in the standard long division algorithm demands a very different way of thinking and reasoning. It utilises a different vocabulary and a different aspect of place value: it involves a shift from quantity value to column value.

The patter associated with the standard algorithm goes something like:

24 into 5 doesn't go. 24 into 56 goes twice. Two 24s are 48, so write 48 under the 56 and subtract to leave 8. Write the 2 on the top line above the 6 of 560, and bring down the zero of 560 to make 80. Three 24s are 72. Write the 72 under the 80 and the 3 on the top line above the zero of 560. Subtracting 72 from 80 leaves 8, so the answer is 23 remainder 8.

The resulting written work looks almost exactly like that in calculation in blue above (except that there would be no zero after 48). However, the reasoning, the place value interpretation, the concepts underpinning the procedures and the accompanying patter are very different indeed. This suggests that there is not a particularly smooth transition from the language and reasoning associated with chunking to the language of the standard algorithm.

Conclusion

An Ofsted report on the teaching of calculation in primary schools (Ofsted, 2002) states the following:

'*However, at Key Stage 2 they [teachers] often overlook the importance of linking pupils' mental strategies to the introduction of expanded and compact written methods.*' (p3)

This belief in a natural progression permeates many NNS and PNS publications, QCA documents and HMI reports. However, I think I have shown in this series of articles that this progression is not as natural as it appears to be, and that more thought needs to be given – and more research carried out – concerning the seamless links suggested in this guidance paper.

Moreover, there is too much emphasis in the document on advancing children to compact methods for all four basic operations as quickly as possible. For the vast majority of children it would be more useful to focus particularly on algorithms that, unlike compact algorithms, have in-built variability that allows for the important principle of differentiation. The following algorithms fit neatly into this category: subtraction by complementary addition, multiplication by the grid method and division by chunking.

Another important aspect of the discussion is that often what appears to be logically or mathematically sound is not necessarily always pedagogically sound. Many of the recommendations in this guidance paper are made from the perspective of the experienced, mathematically-literate adult, without taking into account the available research findings about how children develop calculation strategies and learn written procedures. This situation obtains particularly with reference to the final step in the recommendations for each of the four basic arithmetical operations, when children have to complete the progression to the compact (ie, 'standard') method. In each case this step involves a major shift in the way the digits in the numbers are interpreted: a shift from treating them as quantities to treating them as digits in columns. Work needs to be done on ways of helping children to make this important step.

In a seminal article written almost 30 years ago, Plunkett (1979) argued that the reasons for teaching standard algorithms were out of date then, and that their use led to:

'*. . . frustration, unhappiness and a deteriorating attitude to mathematics.*' (p4)

Plus ça change, plus c'est la même chose!

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The four articles in this series are available as an e-book from the ATM website: www.atm.org.uk/buyonline.

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