# DECONSTRUCTING CALCULATION

## PART 1: ADDITION

In the first of several articles, Ian Thompson deconstructs the primary strategy's approach to written addition.

In the next article I shall consider the primary national strategy's recommendations for the teaching of subtraction.

The aim of this series of four articles is to look critically, and in some detail, at the primary strategy approach to written calculation, as set out on pages 5 to 16 of the 'Guidance paper' ***Calculation***. The underlying principle of that approach is that children should use mental methods whenever they are appropriate, whereas for calculations that they cannot do in their heads they should use an efficient written method accurately and with confidence.

It is interesting to note that in an earlier version of this document that was available in August 2006, 'efficient written methods' were described as 'standard methods'. There is no doubt that the change in terminology was partially due to the mathematics education community's response – forcefully expressed in a ***TES*** article entitled *Outrage at return to 'dark ages',* which discussed the extent to which some senior academics, numeracy consultants and practising teachers were angry about the government's proposals that all children should be using traditional standard methods of calculation for the four basic operations by the time they left primary school. The article concluded with a response from Tim Coulson, the then director of the mathematics section of the primary national strategy, in which he stated categorically that his team would be addressing the concerns expressed. However, all that happened was that the terminology used to describe the recommended algorithms was changed from 'standard' to 'compact', 'efficient' or 'column'. No modifications were made to the actual written methods: they were still the standard algorithms with a different name. So much for consultation! (See Thompson, 2007.)

The strategy's approach to addition is divided into four stages: the empty number line; partitioning; expanded methods in columns; and column methods (originally 'standard methods'). This progression matches the original NNS approach of counting → mental → jottings → expanded written → compact written, but unfortunately, like the NNS, shows a misunderstanding of the purpose of the empty number line (ENL). The Dutch, who developed the ENL, never envisaged it as a link between mental and written strategies, but rather as a tool to support mental calculation. It provides a physical, and then later, a mental model for such calculations.

### Stage 1: The empty number line

In the example below, taken from the Guidance (*fig 1*), notice that only one of the two numbers to be added has been partitioned (known in Holland as the N10 strategy) (see Rousham, 2003). If both numbers are partitioned (the '1010' strategy), you cannot make use of an empty number line (try it!).

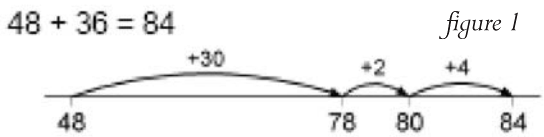


*figure 1*

Written methods, on the other hand – both expanded and compact – involve treating the ones separately from the multiples of ten; ie, partitioning both. This suggests that there is actually no logical progression from ENL use to expanded or contracted written methods, as they are based on conceptually different procedures. The Dutch are well aware of this, and so, after children are considered competent at mental calculation, they spend some time on giving the children practice at using 'double partitioning' (the '1010' strategy) in a range of contexts, before introducing written algorithms that employ such a strategy.

## Stage 2: Partitioning

At this stage children are expected to record their mental strategies horizontally using both single (N10) and double (1010) partitioning:

$47+76=47+70+6=117+6=123$

$47+76=40+70+7+6=110+13=123$

(Unfortunately, these examples disregard the strategy's own insistence that you 'put the larger number first' and have only one equals sign per line!) The focus at this stage should really be just on the second example. The next step involves writing partitioned numbers under one another:

$$\begin{array}{rcl} 47 & = & 40+\ 7 \\ \underline{+76} & & \underline{70+\ 6} \\ & & 110+13 = 123 \end{array}$$

However, there is no acknowledgement that this written strategy builds on the '1010' (double partitioning) strategy and not on the one they have been using for the ENL.

## Stage 3: Expanded method in columns

This stage builds on the example above. We are informed that children should initially 'add the tens first' (*fig 2*) and then, as they gain confidence, they should 'add the ones first' (*fig 3*).

A question that comes immediately to mind is: 'Why add the ones first, when all the research on mental calculation suggests that, left to their own devices, children will start from the left and add the multiples of ten first?' No doubt the answer is that it is to prepare the children for right to left addition in the column method; ie, the standard algorithm.

In a link between stages 3 and 4 the document states that:

> *The expanded method leads children to the more compact method so that they understand its structure and efficiency.* (p.6)

My own research (Thompson, 2002) suggests that the aspect of place value underpinning mental calculation methods and informal written procedures is different from that which underpins the standard (or 'column') written algorithms: the former methods involve 'quantity value' (where 56 is interpreted as *fifty plus six*), whereas the latter procedures involve 'column value' (where 56 is interpreted as *five in the tens column and six in the ones column*). This research would appear to question the validity of the quotation above.

## Stage 4: The column method

This stage introduces 'carrying' (*fig 4*).

The document states that: *Carry digits are recorded below the line, using the words 'carry ten' or 'carry one hundred', not 'carry one'.* (p.6)

This suggests that, as in earlier NNS recommendations, children are expected to refer to the actual value of the digits when performing this calculation: they should say *forty plus seventy equals one hundred and ten*. This is perhaps feasible when adding 2-digit numbers, but becomes much more cumbersome with the addition of 3-digit numbers. The following example involves 'two carries' (*fig 5*).

Trying to refer to the actual value of the digits makes it much more difficult with numbers of this size. After saying *6 add 8 equals 14, put down the 4 and carry the 10*, we write 1 (ie, not a 10) under the 6 and the 5. The next step is to say *60 add 50 equals 110, and 10 more makes 120* – but there is the possibility of an error at this point, as the 10 to be added has been written as a 1 – albeit in the tens column. However, assuming that we perform the calculation correctly and get 120, the next problem is 'Where do we write the three separate digits?' The official answer has to be: *put the 20 as a 2 next to the 4 in our answer (or 'in the tens column'); ignore the zero and put the 100 as a 1 under the 3 and the 4 whilst saying 'carry one hundred'*... No doubt the reader will find this procedure somewhat confusing. This is because, in terms of the discussion of place value above, we are shifting backwards and forwards between 'quantity value' and the more conceptually difficult 'column value'.

I would argue that column methods – being extremely compact – inevitably conceal much of what is actually going on in the calculation. They summarise several steps involving commutativity, associativity and distributivity, whereas, because they contain more detail, non-standard methods record the successive stages of the calculation, thereby allowing children to keep track of where they are and enabling them to ascertain more easily where they have gone wrong if the answer is incorrect. I would therefore question the wisdom of attempting to teach the column method to all primary children, given that the expanded 'front-end' method of addition is more easily understood because it builds on the 'double partitioning' method used by most young children for mental calculation. Like the more difficult standard algorithm, it is also generalisable to the addition of larger numbers and decimals.

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$$\begin{array}{r} 47 \\ \underline{+76} \\ 110 \\ \underline{13} \\ \underline{123} \end{array}$$

*figure 2*

$$\begin{array}{r} 47 \\ \underline{+76} \\ 13 \\ \underline{110} \\ \underline{123} \end{array}$$

*figure 3*

$$\begin{array}{r} 47 \\ \underline{+76} \\ \underline{123} \\ {\scriptstyle 1\ } \end{array}$$

*figure 4*

$$\begin{array}{r} 366 \\ \underline{+458} \\ \underline{824} \\ {\scriptstyle 1\,1\ } \end{array}$$

*figure 5*

**References**

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Rousham. L. (2003) The empty number line: a model in search of a learning trajectory. In I. Thompson, *Enhancing Primary Mathematics Teaching*, Open University Press, p29-39

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