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An investigation of the relationship between young children's understanding of the concept of place value and their competence at mental addition

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1 Background

The aim of this project was to undertake a study of what is often described as the most important and fundamental concept in early number work, namely, the concept of place value. In one of the most influential books in the UK on the teaching of mathematics to young children, *Primary Mathematics Today*, Williams & Shuard (1976) argue that:

> *...as soon as numbers greater than ten need to be written, the first introduction to the structure of our notation has to be made.* (p. 120).

They then recommend a range of grouping and two-column abacus activities that are designed to prepare for the introduction of written notation. To a great extent this has been the approach recommended by almost every commercial mathematics scheme produced in this country over the last 25 years.

Findings from a Nuffield-sponsored project (Thompson & Smith, 1999) show that when adding or subtracting two-digit numbers mentally young children appear not to make use of the column-based interpretation of place value alluded to in the above paragraph. This evidence would appear to raise questions about the appropriateness of the traditional approach recommended for the teaching of place value, particularly given the current emphasis in the National Numeracy Strategy's *Framework for teaching mathematics from Reception to Year 6* (DfEE, 1999) on the teaching of mental calculation rather than formal written methods before Year 4.

2 The Research Question

The main research question asked was: what is the nature of the relationship between young children's understanding of the concept of place value and their ability to successfully perform two-digit calculations mentally?

3 Methodology

3.1 Introduction

The study took the form of a series of one-to-one interviews with a stratified sample of 144 children in Years 2 to 4 from eight primary schools. In the interviews the children were asked to complete a range of practical and written graded questions related to place value. These were considered to be commensurate with the age and ability of the children. They were also asked to calculate mentally with two-digit numbers, after which they were invited to describe the strategy that they had used to generate their solutions. A semi-structured interview schedule was designed in order to help secure consistency in the process, and the interviews were tape-recorded for later transcription and analysis.

3.2 Research Design

- A pilot study in one school provided a trial run for the interviewers; enabled a decision to be made as to which year groups might generate the most useful information; enabled the researchers to select the most appropriate questions; and helped identify potential problems. This pilot study led to the development of an interview schedule that involved nine questions probing different aspects of numerical understanding (see Appendix 1).

- An opportunity sample from three Local Education Authorities of eight schools representing a variety of social backgrounds was made, and six children from each of Years 2, 3 and 4 were interviewed in all eight schools.
- Class teachers were asked to select children from three different groups: those whose attainment in number could be considered to be below average; those whose attainment was deemed to be of average standard, and those who were performing at an above-average level. National Curriculum Test results and the teachers' own assessments of their children's current working level were used to select a boy and a girl from each of these groups in each school year.
- Pupils were interviewed individually following the agreed schedule, and these interviews were tape-recorded. The protocols were then transcribed and analysed in a variety of ways commensurate with the research question described above.

4 Results

After responding to a simple 'warm-up' question 'Which is bigger 82 or 59?' (answered correctly by 98% of the sample) the children were asked a further eight questions. This report will focus in some detail on just four of these – questions 6, 2, 9 and 8 in this specific order.

4.1 Question 6

Can you read this number to me? (Show card with 16 written on it).
Please take 16 cubes out of the box.
Can you show me with the cubes what this part (6) *of the number means?* (Circle the 6 with the back of a pen).
Can you show me with the cubes what this part (1) *of the number means?* (Circle the 1).

Several researchers (Kamii, 1986; Ross, 1989; Hiebert & Wearne, 1992; Price, 1998) have reported the findings of studies with children of different ages and abilities on this question - often called the 'face value' task. Researchers have sometimes used slightly different question structures, but the common features are the following:

- a child is asked to read a two-digit number;
- she is then asked to count out that number of objects;
- the researcher then draws a ring around the units digit of the number in front of the child and asks her to use the objects to show what this particular part of the number means;
- the process is then repeated for the tens digit.

The purpose of the question is to ascertain whether the child understands that, for example with 25 bricks, the 2 stands for twenty bricks rather than just two. Ross (1989) tested 60 American children, 15 from each grade from 2 through 5, and the question she asked each child was: *Does this part of your 25 have anything to do with how many sticks you have?'* She reported that, overall, 26 (43%) of the 60 children were successful in showing that the 2 in 25 represented twenty of the sticks, but that it was not until grade 4 (Y5) that over half of the children in the year group demonstrated this.

Kamii (1988) has argued that the results from the various research projects in America that have used this question suggest that the proportion of children who say that the 1 in 16 means ten is generally 0% at the end of 1st grade (Y2), 33% at the end of 3rd grade (Y4) and 50% at the end of 4th grade (Y5). Confirmation of one aspect of Kamii's assertion comes from Hiebert & Wearne (1992), who interviewed 153 American children in grade 1 and found that none of them showed that the 1 in 16 represented 10 objects.

On a different continent, Price (1998) interviewed 16 Australian Year 3 children and found that 7 (44%) of them said that the 2 in 24 represented 20 pop sticks. After 10 sessions working with materials modelling two- and three-digit numbers (base-ten blocks for one group and computer software for the other), only one extra child gave the correct answer. These data and the results of this study are shown in Table 1.

Grade/Year	% **Ross** (USA)	% **Kamii** (USA)	% **Price** (AUS)	% **This study** (ENG)
1/2	-	0		54
2/3	20		44	77
3/4	33	33		79
4/5	53	50		
5/6	67			
Mean	**43**	*	44	70

Table 1. The results of four researchers on the 'face value' task

*As these data come from different sources it is not possible to calculate the mean percentage

The results of the successful Y2, Y3 and Y4 children in the study reported here were, respectively, 26 (54%), 37 (77%) and 38 (79%): results well in excess of those produced by the children in the other studies. These inflated percentages are not too easy to explain, given that the grade 1 results are as good as Ross's grade 4, and the grade 2 results are 10% better than those of Ross and Kamii's grade 5 children. It is also important to note that the language used in the researchers' interactions with the children was almost identical, and that both the English and American researchers used an identical number of objects in their studies.

Kamii (1985) has argued that children develop their natural ability to think logically and 'reinvent arithmetic' via the mental activity that takes place in social interaction. She developed what she called a 'new way' of teaching place value and double-column addition based on Piagetian principles and consisting of games in kindergarten and first grade, followed by games and discussions in second grade (Kamii, 1988). The approach involved no working with bundles of 10, no drawing circles round sets of 10 in workbooks and no manipulating base 10 blocks. By second grade the children answered written calculations (set out vertically) using a range of invented idiosyncratic mental strategies. These included counting on, near doubles, bridging and partitioning, and were discussed and evaluated initially by the teacher and then later by the children. No set procedures were taught, and Kamii found that by the end of second grade **66%** of the children (compared with Ross's 20% and Price's 44%) said that the 1 in 16 meant ten.

Several factors that might account for the **77%** success of second graders reported in this study are listed below.

- The percentages of successful grade 3 children in both American studies are identical (33) and are very close in grade 4 (53 : 50). As American children start school one year later than English and Australian children it might be expected that children from the latter countries would perform better in the early grades.

- Kamii's (1988) interactive and less formal approach to calculation, involving discussion and sharing of methods, produced a substantial increase in the percentage of successful children in grade 2. In England, the approach to calculation advocated in the National Numeracy Strategy (NNS) takes Kamii's approach further, recommending substantial emphasis on mental methods from the very start; stressing the need for interactive whole-class teaching, frequent teacher and pupil discussion of mental calculation strategies and a recommended delay in the introduction of formal written calculations until Year 4 (grade 5).
- The NNS also advocates the frequent use of Gattegno tables and place value cards (Appendices 2 and 3) for children to display their answers as a whole class to calculations given by the teacher in the 'oral/mental starter' section of the recommended 'three-part lesson'. These tables and cards reinforce the constituent components of 2- and 3-digit numbers because children have to put together, say, 400, 50 and 3 in order to make 453.
- When written methods are introduced in the NNS children are initially encouraged to use jottings and later to set out their calculations horizontally, thereby inviting them to extend their left to right mental strategies to calculations given in written form. This relates more closely to mental methods than does Kamii's (1988) early introduction of a vertical layout.

Clearly more research needs to be done to investigate UK children's apparent superiority on questions of this type.

4.2 Question 2

(a) *What is 25 plus 23?* (Show card with 25 + 23 written on it).
(b) *Tell me how you did it.*

Eighteen children (12%) were not able to provide an answer to this question and 13 (9%) gave an incorrect answer. The remaining 113 (79%) gave the correct answer. This included 10 children (7%) who changed their original response during the process of describing their strategy. Of the 18 children who failed to give an answer, ten were from the lowest-attaining Y2 group, four were from the middle Y2 group and the remaining four were from the lowest-attaining Y3 group. Similarly, the 13 children offering incorrect answers were generally from the lowest-attaining groups in Y2 and Y3 or the middle group from Y2. The only exception was a Y4 child from the middle group who had no doubt been taught the useful strategy of doubling, but who incorrectly adapted it to find 25 + 23 by doubling 20, 5 and 3 and adding 40, 10 and 6 to get 56.

Strategy use on Question 2

Those children who gave incorrect answers made use of just two strategies: counting-on (4 instances) and partitioning (5 instances). It was impossible to decide the strategies of the other four children. The 113 children who gave correct answers to the question made use of six different strategies: counting-on, manipulating digits, partitioning, mixed methods, sequencing and near doubles. In Table 2 the notation given in brackets is that used in the research literature.

Strategy	Numbers of successful users
Counting-on	3
Manipulating digits	14
Partitioning (1010)	82
Mixed method (1010S)	3
Sequencing (N10)	6
Near doubles	5

Table 2. Strategies used by successful children to calculate 25 + 23

For clarification 25 + 23 has been calculated below using three of the methods mentioned in Table 2:
Partitioning (1010): 20 + 20 = 40; 5 + 3 = 8; 40 + 8 + 48;
Sequencing (N10): 25 + 20 = 45; 45 + 3 = 48;
Mixed method (1010S): 20 + 20 = 40; 40 + 5 = 45; 45 + 3 = 48.

The first five of the strategies in Table 2 have been discussed in some detail by Thompson and Smith (1999) who classified them, respectively, as Levels 1 to 5 in terms of 'levels of sophistication'. The sixth strategy, near doubles, was not included in the 1999 study as it is only useful in special cases where the numbers involved are close together and the relevant doubles fact is known. The five children who correctly used this strategy were all from the average or above-average groups in Y3 or Y4. Counting on was used by four children who gave incorrect answers and three who gave correct answers. It was also observed that three of the children who did not give an answer appeared to be attempting to use counting-on. This means that just 30% of the children using this strategy in this study were successful.

The 1010, N10 and 1010S methods can be grouped together as general partitioning strategies as they require at least one of the two addends to be split into a multiple of ten and a single-digit number before or during the addition process. This grouping of strategies gives a total of 91 children (63%) who successfully used a method that involves the partitioning of one or more of the two-digit numbers when finding their sum (Table 3).

Year	Successful children
2	21 (44% of year group)
3	35 (73% of year group)
4	35 (73% of year group)
Total	**91 (63% of sample)**

Table 3. Children who successfully used partitioning to find 25 + 23 mentally

4.3 Question 9

Show picture of odometer and discuss.
I have just looked at the dial in my car that tells me how many miles I've driven, and it shows this number (06142).
What do you think it will show when I've driven another mile?
If the dial had shown this (06299) *what would it have changed to?*

This question was adapted from one used by Ward (1979) and by Brown (1981). The children were shown a drawing of the fascia of a car. The interviewer talked to each child about their experiences of being in a car and discussed the purpose of the speedometer and the odometer (without using these words), ensuring that each child understood the principle of how the numbers changed as the car travelled. Having set the scene the interviewer then asked the children the two parts of the question. The first part was given as a warm-up activity to the main question where the odometer reading was 06299 miles.

In this study the correct answer was given by only 35 (24%) of the children, despite the interviewer's attempts to familiarise them with the context of the problem. Table 4 shows the number of children from each attainment group in each school year whose answers were correct (both 06300 and 6300 were accepted as correct). Between them the children produced a wide variety of incorrect responses, ranging from those that were almost correct – 6200 – to those whose logic was difficult to follow – 278 105. There were actually 31 different incorrect answers. These results (Table 4) suggest that less than one quarter of the children interviewed appear to be aware that 6300 comes after 6299.

Year	Successful children	
2	3	8%
3	11	23%
4	21	44%
	Total **35**	Mean **24%**

Table 4. Number of children giving correct answers to question 9

Brown (1981, p. 49) considers this question to have acted as a useful test of children's grasp of place value. Comparison of the findings of this study with the Ward and the Brown data cannot really be made because of the different age groups. However, whilst appreciating that there were different sample sizes and selection procedures involved in the three studies, combining the results does actually generate an interesting pattern. Figure 1 shows the percentage of each age group giving correct answers to very similar questions in very similar contexts in the three projects, and suggests that this particular question could be considered to have good discrimination value.

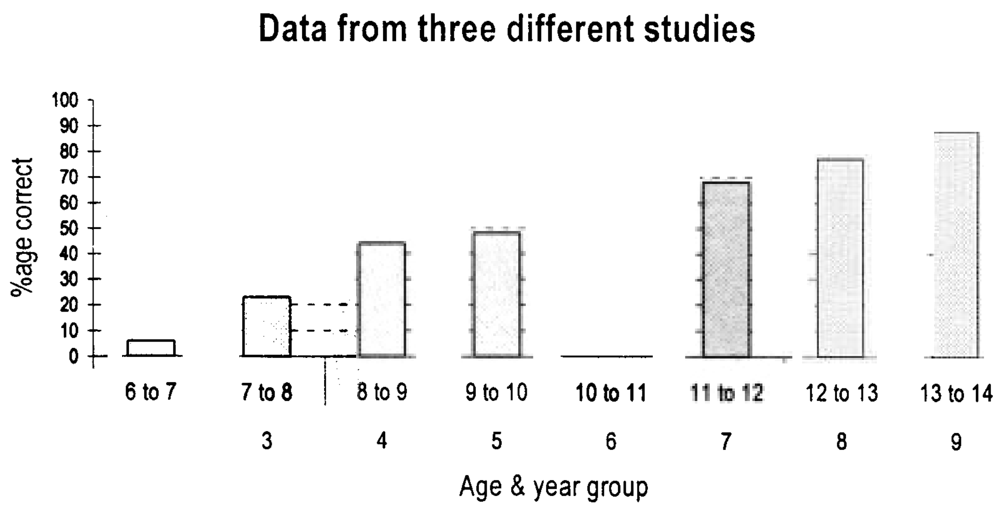

Figure 1. Data from three different studies

4.4 Question 8

8 (a) Use a 100s, 10s and 1s board.
Put 3 cubes in the 10s column and four in the ones column and ask:
What number does this represent? (Explain/discuss if they get it wrong – 'Each of these cubes represents 10 so what does that make, and...').
Remove the three cubes from the tens column, and point to the remaining cubes.
These cubes are in the ones column and so we say they represent 'four'. I'm moving the cubes to the next column (say this as you do it)... *Can you tell me how the value of the cubes has changed?*
8 (b) For those pupils whose answer gives no indication of the fact that the value has increased by a factor of ten ask the following question: *How many times bigger is it now?*

This question is a modified version of a question used by the Assessment of Performance Unit (APU, 1982). An initial familiarisation activity was given which involved the interviewer in placing three cubes in the column with 10 at its head and four cubes in the column headed 1. Children who were unable to say what number was represented by the cubes were then engaged in a discussion that usually concluded with their agreeing that together the two cubes represented 34. The main question was then asked.

As Table 5 shows, only 10% of the whole sample gave an answer which suggests that they appreciated that moving the cubes into the next column to the left was equivalent to multiplying by ten. A further 18% gave such a response after being asked the follow-up question *How many times bigger is it now?* This was despite the fact that children had earlier been asked to calculate 8 x 10, and had then been asked *What happens when you multiply by ten?* One hundred and twenty-two children (85%) gave 80 as their answer to the first part, and not a single child gave an answer to the second part that even vaguely hinted at the movement of digits.

Year	8a	8b (follow up)
Y2	1 (2%)	6 (13%)
Y3	3 (6%)	14 (10%)
Y4	10 (21%)	21 (44%)
Total	**14 (10%)**	**41 (28%)**

Table 5. Number of children giving the correct answer to the two parts of question 8

4.5 Combining the results from questions 8 and 9

Questions 8 and 9 were chosen because they were deemed to assess children's understanding of two important aspects of place value. Question 8 involves the realisation that moving the digits one place to the left is equivalent to making a number ten times bigger, whereas question 9 involves an appreciation of the effect on other columns when one more is added to a column containing a nine. The questions were also used because they had been tried and tested by other researchers (Ward, 1979; Brown, 1981).

It is argued here that any child who might be considered to understand place value should appreciate both of these principles in addition to others that have not been included in this study. To facilitate communication in the ensuing discussion it was decided to categorise the children into four 'levels of understanding' based on the their success on these two particular questions. Those children who correctly answered both questions 8a and 9 were considered to appreciate both of the principles outlined above, and their understanding was classed as 'excellent', whereas those who were successful on 8b and 9 were categorized as having a 'good' understanding. Those children who correctly answered either of questions 8 or 9 but not both were deemed to have a 'fair' understanding, and those children who were unsuccessful on both questions were considered to have a 'poor' understanding. The children who successfully calculated the answer to 25 + 23 in their heads using a partitioning strategy were called 'calculators'. This classification system was then used to categorise the 91 successful calculators (Figure 2).

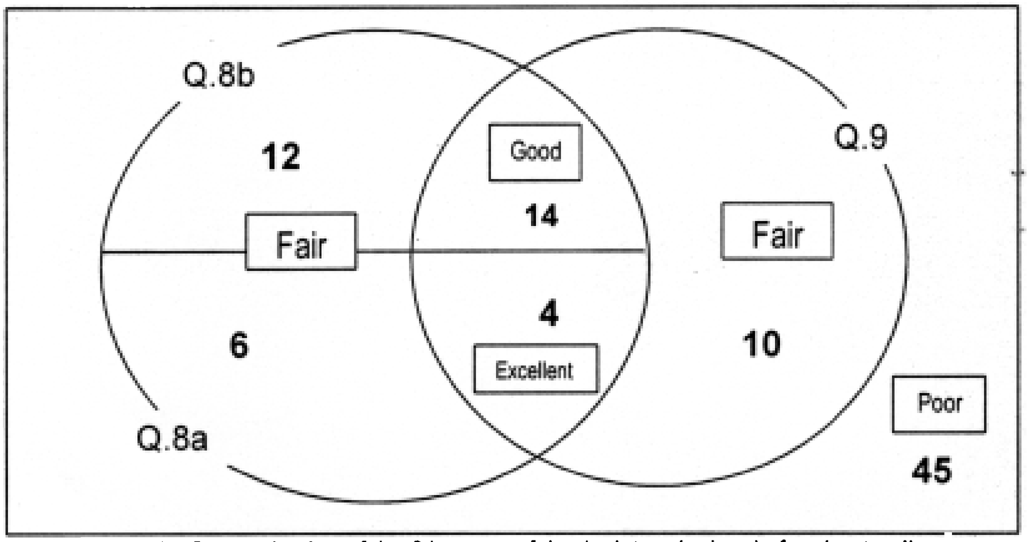

Figure 2. Categorisation of the 91 successful calculators by level of understanding

Figure 2 shows that only four children (4%) performed well enough to be classified as 'excellent' in their understanding of place value with 14 (15%) achieving the 'good' level. A further 28 (31%) answered just one of the questions successfully and so appear in the 'fair' category, and the remaining 45 (49%) were graded as 'poor' in their understanding. This means that half of the successful calculators answered both questions 8 and 9 **incorrectly**.

This information has been broken down by year group in Table 6 and a 'fair or better' category added by combining the 'excellent', 'good' and 'fair' results. Consequently, this extra column includes those children who were successful in answering one or more of the two questions correctly.

	Excellent	Good	Fair	Fair or better
Y2	0	2	2	4
Y3	0	4	13	17
Y4	4	8	13	25
Total	**4** (4%)	**14** (15%)	**28** (31%)	**46** (51%)

Table 6. Levels of place value understanding of the 91 calculators

This table shows, as might be expected, that the results improve as children get older. It is also the case that the four children in the excellent category were not only all from Y4 but were also from the 'above average' group, suggesting that an understanding of place value is attained quite late in a primary school child's development. Another way of analysing these findings is to consider those calculators who gave *incorrect* responses to both of the place value questions (Table 7).

	Calculators	Poor
Y2		
Y3		
Y4		
Total		

Table 7. Number of calculators with 'poor' understanding of place value

Table 7 shows that 17 of the 21 successful calculators in Y2 were in the 'poor understanding' category. What this means is that four fifths of the Year 2 children who were able to use partitioning correctly to find 25 + 23 gave incorrect answers to both of the key place value questions. In Y3 and Y4 the number of calculators was identical but the number with poor understanding in the older group was half that of the younger, supporting the argument that place value understanding does indeed develop slowly over several years.

5 Discussion

The Venn diagram in Figure 2 showed clearly that only **four** of the 91 successful calculators could be said to have shown an excellent understanding of place value by dint of the fact that they successfully answered both question 8a and 9. This obviously means that 87 (96%) of them were considered not to have achieved this level of understanding, and yet they were still able to perform a two-digit calculation successfully using partitioning and recombining – a procedure that would appear to depend substantially on a thorough understanding of place value. But if these children do not understand place value then what is it that they do understand?

The ability to perform such calculations must require a basic understanding of certain aspects of the structure of the number system and specific relationships within that structure. The findings reported earlier on the 'face value' task (question 6: 'the 1 in 16 stands for 10 cubes') give the impression that the children in this study have a very good understanding of place value compared with children of a similar age in other studies in other countries (70% success rate compared with 44%), and yet only 4% of them successfully answered both question 8a and question 9. There is clearly an important difference between the aspects of place value probed by these two questions and those probed by the 'face value' task.

The results of this study show that it is possible for children to know that 16 comprises one ten and six ones without their being aware of the column structure of the notation system used for written numbers. The children in this study are likely to have learned about 16 being ten and six from the substantial work on partitioning and recombining of numbers recommended throughout National Numeracy Strategy publications. They will probably have recognised that 16 seen as 10 and 6 is an easy partition to remember, and will have developed an implicit appreciation of the effect on the written form of a number when 10 is added to any single digit number. They are also likely to have learned that the 2 in 25 means 'twenty' because of the way the number is said or read. However, it does not necessarily follow that these same children will be aware that the 2 is in the tens column and therefore means 'two tens'. In this study only 14 (15%) of the 91 successful calculators used a phrase like 'two and two make four' when adding 25 and 23, whereas the remaining 77 (85%) said 'twenty and twenty make forty'. Given that half of these 14 children came from just two different schools, it seems likely that they had been taught a formal calculation algorithm. It is important to note that not a single child explained their working as 'two tens and two tens make four tens'.

Thompson (2000) has demonstrated that children's informal mental and written calculation strategies, as described in the literature, rely substantially on what he calls *quantity value*: the understanding that a two-digit number such as 47 can be partitioned into 'forty' and 'seven' and that the partitioned parts can be operated upon separately. The research reported here would appear to strengthen his argument that there is an important distinction to be made between knowing that 73 is 'seventy' and 'three' and knowing that it is 'seven tens' and 'three units'. This distinction lies at the heart of his argument that what we have up to now called place value should be seen as comprising two separate concepts: *quantity value* and *column value*. The findings of this study suggest that an understanding of the former develops before the latter, and lends support to the argument for delaying the teaching of tens and units – column value – until later in the curriculum. According to Thompson (1999) this teaching need not take place until such time as children are to be taught the standard algorithms for the basic operations. It is interesting to note that the Netherlands was the most successful European country in the primary mathematics section of the Third International Mathematics and Science Survey (Keys et al, 1996), and yet the Dutch make little or no mention of place value in what might be considered their equivalent of our National Curriculum (Freudenthal Institute, 2001).

When children are adding twenties in this study their focus of attention is the oral/aural aspect of number rather than the written aspect. Recent psychological research suggests that these might be processed in different parts of the brain. Donlan (1998) writing about issues in current 'brain-based' research argues that

> '*...there is general acknowledgement that numerical information processing, in the adult at least, involves verbal and non-verbal systems that may operate independently.*' (p. 257).

The research reported here suggests that this may well also be true for children. The implication is that teachers and children need to spend a substantial amount of time developing links between these verbal and non-verbal systems.

Ross (1985) has argued in her stage model of place value understanding that children's appreciation that the tens digit represents sets of ten objects is contemporaneous with the acquisition of the concept of the whole being the sum of the parts. Sinclair's (1992) data suggest that the latter is acquired well before the former, and the results of this study add weight to her argument in that 63% of the sample correctly added two two-digit numbers using partitioning: a procedure that demands an appreciation not only of the fact that the whole is the sum of the parts, but that the separate parts can be added and a new whole constructed from these new parts. Of these 91 successful calculators only 4% were in the 'excellent understanding of place value' category. The weakness in Ross's (1985) argument probably stems from her failure to distinguish between knowing that the 2 in 23 stands for 'twenty' and knowing that it stands for 'two tens', i.e. between 'quantity value' and 'column value'.

In 1985 Hilary Shuard stated that:

> *...in the present curriculum the introduction of ideas of place value is probably postponed for too long for many children...* (p. 114).

The findings of this study suggest that in the present curriculum ideas of place value, *at least with regard to the aspect of column value*, might be introduced too early for many children. Further research should investigate the possibility that the teaching of column value so early, as is still done in many English and American schools, contributes towards both countries' relatively poor performance in the number section of international mathematics surveys.

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SHOW CARD

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[illegible]

SHOW CARD

[illegible]

[illegible]

Show card with 16 written on it

[illegible]

Can you show me with the cubes what this part (6) *of the number means?*

Circle the 6 with the back of a pen

[illegible]

Circle the 1

7

Only for those who get question 6 correct

Count the number of cubes here

(6 lots of 4 cubes joined together and 2 loose cubes which are placed in front of the 6 lots of 4)

SHOW CARD WITH 26 ON AND SAY *Yes, this is how many there are. Can you show me with the cubes what this part of the number means?* CIRCLE THE 2

8

(a) Use a 100s, 10s and 1s board

Put 3 cubes in the 10s column and four in the ones column and ask: *What number does this represent?*
(Explain/discuss if they get it wrong – 'Each of these cubes represents 10 so what does that make, and...')

Remove the three cubes from the tens column, and point to the remaining cubes.
These cubes are in the ones column and so we say they represent 'four'. I'm moving the cubes to the next column (say this as you do it)... *Can you tell me how the value of the cubes has changed?*

(b) For those pupils whose answer gives no indication of the fact that the value has increased by a factor of ten ask the following question: *How many times bigger is it now?*

9

SHOW PICTURE OF ODOMETER AND DISCUSS

I have just looked at the dial in my car that tells me how many miles I've driven, and it shows this number

0	6	1	4	2

Write down what you think it will show when I've driven another mile

If the dial had shown this what would it have changed to?

0	6	2	9	9

Write the answer here.

Appendix 2 A Gattegno table (up to 9999)

1000	2000	3000	4000	5000	6000	7000	8000	9000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

Appendix Arrow cards, Gattegno cards or place value cards